A SURVEY OF LQR CONTROL FOR BALL-ON-WHEEL SYSTEM: SIMULATION AND EXPERIMENT

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Abstract: A ball-on-wheel system is a recently developed model in the field of automatic control. It serves as a simple model, meeting the learning and algorithmic research needs of students. With this model, there are various algorithms available for system control, such as PID controllers, fuzzy PID controllers, and sliding mode controllers, among others. In this paper, we construct a mechanical model for the system. We choose the Linear Quadratic Regulator (LQR) algorithm to design for this system. Simulation and experimental results demonstrate the effective operation of the LQR controller for the inverted pendulum on a cart system. Additionally, tuning experiments indicate that the parameters have been verified and confirmed to be consistent with the theoretical tuning principles of the LQR controller.

Keywords: Ball on wheel; LQR; Balance control.

1. Introduction

In automatic control, algorithms are researched based on experimental and simulated system behaviors. Classic models like inverted pendulum [1], [2] and ball on beam [3], [4] serve as common platforms for research in automatic control. However, it's also essential to expand models to validate algorithms for different types of systems. One such developed model is Ball on Wheel system. In this system, a mechanical setup consists of a wheel driven by a motor encoder to rotate vertically, with a ball placed on wheel. Rotational motion of wheel ensures the ball stays on the wheel. This can be viewed as a single-input-single-output (SISO) system if only position of ball is of interest. Additionally, ball on wheel can be seen as a single-input-multiple-output (SIMO) system if both wheel rotation angle and ball position are considered. Researching and experimenting with basic algorithms on this model aids in standardizing model. Thus, with a verified and appropriate model, successful implementation of algorithms on model helps students understand and apply these algorithms to similar real-world objects, such as balance systems for submarines or vibration-damping systems for high-rise buildings. Furthermore, with simple validated models, researched algorithms contribute to fundamental learning with minimal costs.

In a study [5], a mechanical ball on wheel was built to investigate control both in simulation and experimentation. However, processing utilized DSP, which is expensive and challenging to equip in laboratories. Additionally, a sliding mode control (SMC) algorithm was presented and performed well in the simulation for ball on wheel. However, results were only verified through simulation and not experimentally validated. Therefore, a simple experimental model, capable of successfully validating control algorithms, remains crucial. STM32 is an integrated control board with mid-range cost and widespread popularity [6]. STM32 community support and usage are robust. Thus, developing simple control algorithms like LQR, validated in both simulation and experimentation, is appropriate, and Feedback linearization has been applied to various real-world problems as well as laboratory experiments, such as an electromagnetic system [7], an electromechanical system [8], and motors [9] – [14]. In this study, we construct an experimental ball on wheel model and algorithm research. We apply LQR algorithm to this system because it's a common algorithm in academic activities. Simulation results show the controller operates well. Additionally, experimental results demonstrate feasibility of this algorithm. Some surveys also indicate that parameter tuning in simulation and experimentation aligns with LQR theory.

2. System Modeling

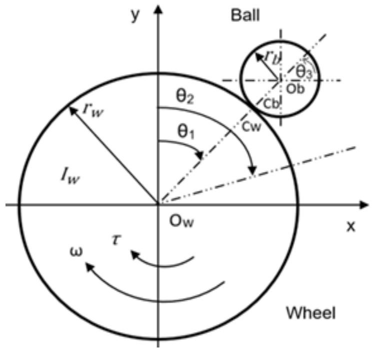

Fig. 1. System modeling of ball on wheel

Tab. 1. System parameters

Parameters	Signs	value
Moment of the inertia of the wheel	I_ω	$1.427\times10^{-3}\ kg-m^2$
Radius of the wheel	r_ω	$0.11\ m$
Mass of the ball	m_b	$0.0569\ kg$
Radius of the ball	r_b	$0.03285\ m$
Resistance of the motor	R_a	$2.2826\ \Omega$
Constant of the motor	K_m	$0.0926\ N-m/A$
Standard gravity	g	$0.981\ m/s^2$

Other papers have demonstrated that the equation, based on the mathematical model (according to reference [5]), has the defined state vector as:

$$x=[x_1\quad x_2\quad x_3\quad x_4]^T=[\theta_1\quad \dot{\theta}_1\quad \theta_2\quad \dot{\theta}_2]^T \tag{1}$$

The state space of Ball on Wheel is written as:

$$\dot{x}=f(x)+g(x)u \tag{2}$$

where $f(x)=\begin{bmatrix} x_2 \\ ax_4+b\sin x_1 \\ x_4 \\ px_4+q\sin x_1 \end{bmatrix}$; $g(x)=\begin{bmatrix} 0 \\ c \\ 0 \\ r \end{bmatrix}$;

$$a=-\frac{2r_\omega K_m^2}{R_a(7I_\omega+2r_\omega^2 m_b)(r_b+r_\omega)};$$

$$b=\frac{g(5I_\omega+2r_\omega^2 m_b)}{(7I_\omega+2r_\omega^2 m_b)(r_b+r_\omega)};$$

$$c=\frac{2r_\omega K_m}{R_a(7I_\omega+2r_\omega^2 m_b)(r_b+r_\omega)};$$

$$p=-\frac{7K_m^2}{R_a(7I_\omega+2r_\omega^2 m_b)};\qquad q=\frac{2gr_\omega m_b}{7I_\omega+2r_\omega^2 m_b};$$

$$r=\frac{7K_m}{R_a(7I_\omega+2r_\omega^2 m_b)};\ ar=cp$$

3. LQR Control

In this study, LQR algorithm is applied. If we consider ball on wheel as a SIMO system, voltage supplied to motor is determined by a single input. Wheel rotation angle and ball position are determined by a multiple output. Structure of LQR controller is used as follows:

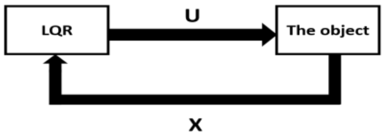

Fig. 2. The block diagram of the LQR control system

In which, u is control signal to object; x are state variables of object; the object is the nonlinear system; LQR controller computes control signal

From the state vector in section 2, we linearize the system at the equilibrium point, with input voltages and the angles theta1 and theta2 = 0, the matrices A and the matrices B can be calculated as follows:

$$A=\begin{bmatrix} \frac{df_1}{dx_1}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_1}{dx_2}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_1}{dx_3}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_1}{dx_4}\Big|_{\substack{x=x_0\\u=u_0}} \\ \frac{df_2}{dx_1}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_2}{dx_2}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_2}{dx_3}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_2}{dx_4}\Big|_{\substack{x=x_0\\u=u_0}} \\ \frac{df_3}{dx_1}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_3}{dx_2}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_3}{dx_3}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_3}{dx_4}\Big|_{\substack{x=x_0\\u=u_0}} \\ \frac{df_4}{dx_1}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_4}{dx_1}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_4}{dx_1}\Big|_{\substack{x=x_0\\u=u_0}} & \frac{df_4}{dx_1}\Big|_{\substack{x=x_0\\u=u_0}} \end{bmatrix} \tag{3}$$

$$B=\begin{bmatrix} \frac{df_1}{du} & \frac{df_2}{du} & \frac{df_3}{du} & \frac{df_4}{du} \end{bmatrix}^T \tag{4}$$

We choose the weight matrix Q and R as follows:

$$Q=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ R=1 \tag{5}$$

The LQR control signal for the system is calculated as

$$u=-Kx \tag{6}$$

With $K=R^{-1}B^TP$, where P is the solution of the Riccati algebra:

$$PA+A^TP-PBR^{-1}B^TP+Q=0 \tag{7}$$

4. Simulation

In our MATLAB simulation, we investigated the variation of the control signal (ut), the position of the ball relative to the reference position (x1), and the deviation angle of the wheel from the reference position (x3) with variations in the parameters of the control matrix Q and R.

4.1. The Control Signal

This study examines the fluctuations in the control signal (ut) concerning alterations in the control matrix R. The findings are succinctly depicted in Fig. 3, providing insight into the impact of varying R values on the control mechanism.

This study investigates the influence of increasing values of the control matrix R (ranging from 0.1 to 10) on the stability and noise resilience of the control signal (ut). Results demonstrate a notable reduction in signal volatility as R values increase, with minimal noise interference observed at R = 0.1. Furthermore, higher R values contribute to enhanced signal stability.

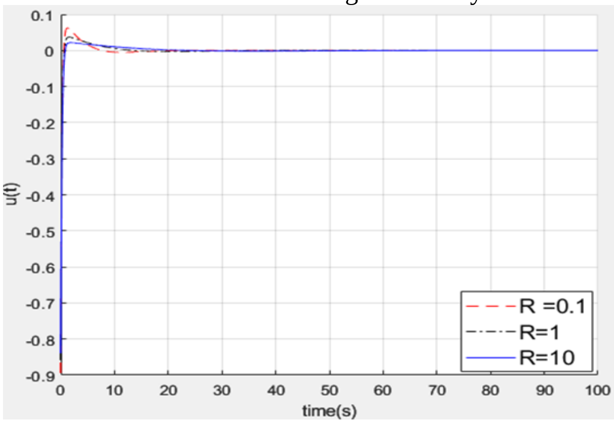

Fig. 3. The control signal

4.2. Ball Position

This study investigates the effect of increasing values of the control matrix Q, specifically the weight q1 associated with the ball position in the state equation of the system. By systematically increasing q1 values (ranging from 1000 to 10,000,000), changes in the ball position are analyzed and presented in Fig. 4.

Results demonstrate a clear correlation between increasing q1 values and a reduction in the initial wheel deviation. Specifically, the deviation decreases from -0.00014 rad (q1 = 1000) to -0.00004 rad (q1 = 10,000,000), indicating improved initial alignment with higher q1 values. Moreover, higher q1 values enhance the stability of the ball position, mitigating noise interference and ensuring smoother trajectory tracking.

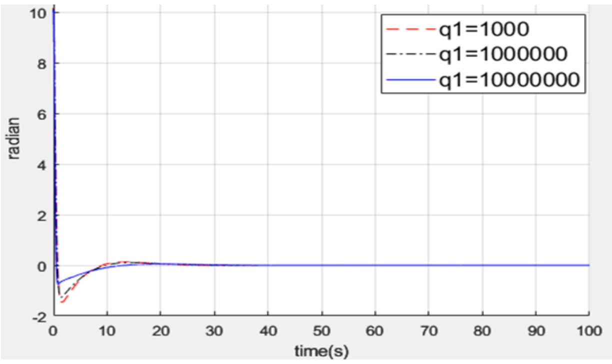

Fig. 4. Deviation angle of the ball

4.3. Wheel Position

This study explores the impact of increasing values of the control matrix Q, particularly the weight q3 associated with the wheel angle in the state equation of the system. Through systematic increments of q3 values (ranging from 100 to 10,000), the variations in the wheel angle are analyzed and presented in Fig. 5.

The results demonstrate a clear correlation between increasing q3 values and a reduction in the time constant of the wheel angle response. Specifically, the time constant decreases from 2.04s (q3 = 100) to 0.82s (q3 = 10,000), indicating faster response dynamics with higher q3 values. Additionally, the initial deviation angle of the wheel decreases from 0.004 rad (q3 = 100) to 0.001 rad (q3 = 10,000), reflecting improved initial stability with increasing q3.

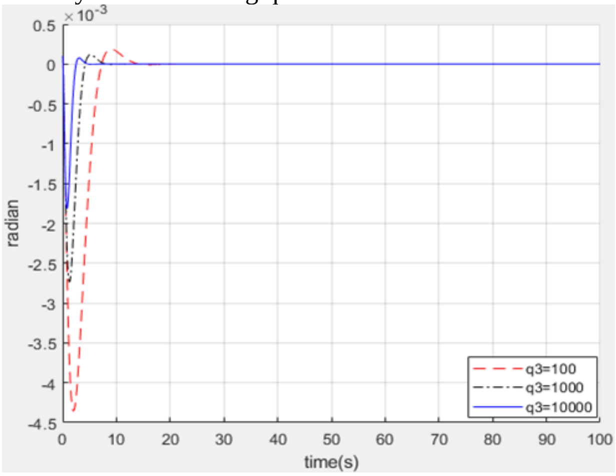

Fig. 5. Deviation angle of the wheel

Control matrices Q and R weights play crucial roles in shaping behavior and performance of dynamic systems. Theoretical frameworks provide guidelines for selecting appropriate weights within these matrices to achieve desired control outcomes. This study evaluates consistency between theoretical expectations and simulated results regarding adjustments in Q and R.

5. Experiment

5.1. System Model

The ball on wheel system model is designed as shown in Fig. 6 and Fig. 7 [15].

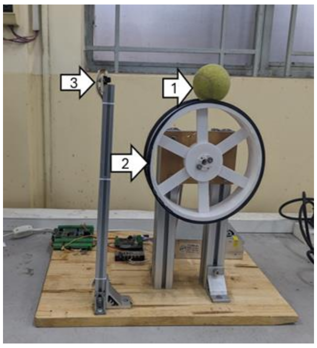

Fig. 6. Ball on wheel viewed from the front

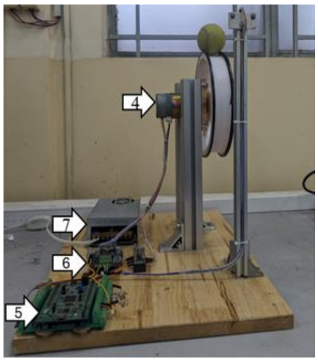

Fig. 7. Ball on wheel viewed from the side

1. tennis ball.
2. the wheel.
3. infrared sensor SHARP GP2Y0A02YK0F.
4. NISCA 24VDC motor.
5. microprocessor stm32f4
6. HI216 power circuit
7. DC 24V source

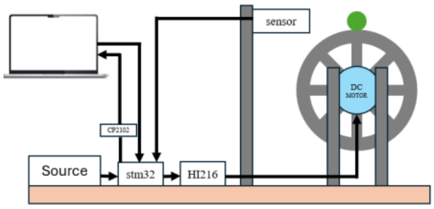

Fig. 8. System block diagram [15]

In the experimental section, we examined the variations in the control signal (ut), the position of the ball relative to the reference position (x1), and the deviation angle of the wheel from the reference position (x3) with variations in the parameters of the control matrix Q and R.. We evaluated the results in comparison to the simulation and theoretical predictions.

5.2. The Control Signal

In this section, we chose the values of the control matrix as R = 0.1 and R = 10. The results are presented in Fig. 9.

This study investigates the impact of adjusting the control matrix R on the amplitude oscillation of the control signal (ut). Experimental results reveal that increasing R values (from 0.1 to 10) lead to a decrease in the amplitude oscillation of the control signal, with a reduction of approximately 2050 at the peak oscillation position. Moreover, higher R values contribute to improved stability of the control signal oscillations.

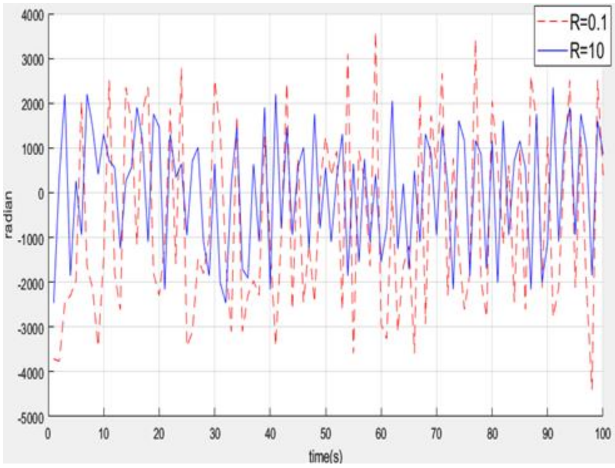

Fig. 9. The control signal

5.3. Ball Position

This section presents an experimental assessment of the real-world deviation angle of a ball from its equilibrium position when altering the weight q1 (representing the ball deviation angle in the system state equation) in the control matrix Q. The study examines q1 values of 10,000 and 100,000, and the results are depicted in Fig. 10.

Experimental results demonstrate that increasing q1 values lead to more stable deviation amplitude, with a reduction of 0.007 radians at the maximum oscillation position. Additionally, the initial deviation angle of the ball significantly decreased from 0.021 radians to 0.008 radians with increasing q1 values. These findings align with theoretical predictions, confirming the efficacy of q1 adjustments in enhancing system stability.

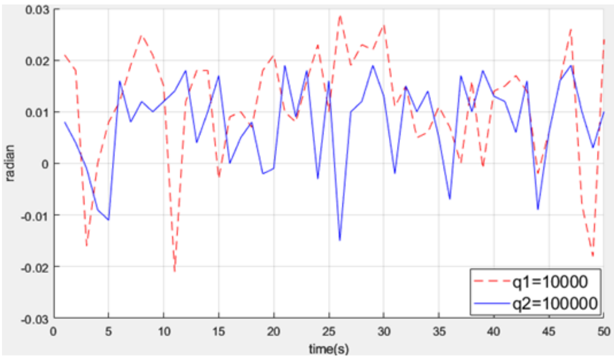

Fig. 10. Deviation angle of the ball

5.4. Wheel Position

This section presents an experimental investigation into the real-world deviation angle of the wheel from its equilibrium position when altering the weight q3 (representing the wheel deviation angle in the system state equation) in the control matrix Q. The study examines q3 values of 300 and 500, and the results are depicted in Fig. 11.

With q3=300, the wheel deviation angle exhibits instability, deviating significantly from the equilibrium position. However, increasing q3 (to value 500) gives oscillation to wheel deviation angle around equilibrium position though it is still unstable due to hardware limitations. Nonetheless, this demonstrates the effectiveness of increasing q3 in managing the wheel deviation angle.

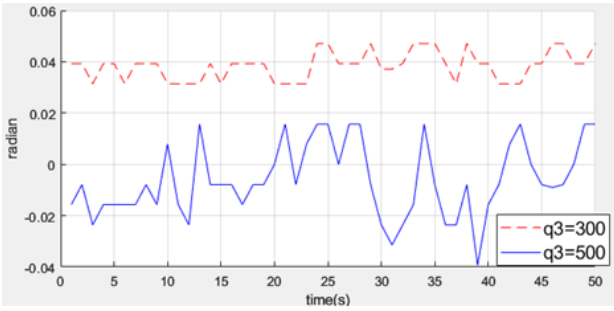

Fig. 11. Deviation angle of the wheel

5.5. Results

The experimental results confirm that increasing the value of control matrix R leads to more stable oscillations of the control signal, reducing amplitude fluctuations. Additionally, variations in control matrix Q weights, particularly q1 and q3, directly influence system state equation components, affecting the ball and wheel deviation angles from the reference position (x1 and x3, respectively). These observations align perfectly with theoretical expectations, highlighting the direct influence of Q and R adjustments on system behavior.

6. Conclusions

This paper presents the successful development and experimental validation of a "ball on wheel" model, utilizing LQR algorithm. Prior to real-world implementation, the model was rigorously verified through both simulation and experimentation. The results demonstrate the effective equilibrium achieved by the ball on the wheel system through the classical LQR algorithm. The proposed "ball on wheel" model serves as a standardized experimental platform, offering an ideal foundation for students to validate subsequent algorithms. It is poised to find widespread application in laboratory settings for educational and research purposes, owing to its low cost, simple structure, and extensive community support.

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8. References

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