A STUDY OF PID CONTROLLER FOR PARALLEL DOUBLE-LINKED ROTARY INVERTED PENDULUM

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Abstract: One-linked Rotary Inverted Pendulum is a classical model in control automation. Many directions of development to increase the complication is operated, such as, putting more link to create two-linked, three-linked... systems. Another direction which is less popular is creating a parallel double-linked rotary inverted pendulum. In this paper, we build a PID controller through investigation the operation of system for balancing this model and use genetic algorithm to find suitable control parameters. Genetic algorithm also show its ability in optimizing PID controller under different generations. These surveys are operated in Matlab/Simulink.

Keywords: parallel rotary inverted pendulum; PID control; genetic algorithm; optimizing controller; balancing control.

1. Introduction

Rotary Inverted Pendulum (RIP) is a basic model in control theory. Quanser creates an educational real model [1]. Many algorithms are researched through this standard system [2]. To make system more complicated for develop controllers for high-order SIMO system, one more pendulum is added to former pendulum to create double-linked RIP **[3]**. Many researches are investigated on this developed model **[4]**. Another less popular way to develop RIP is adding another pendulum on other part of the arm to create parallel RIP (PRIP). Very few researches are considered on this model. In **[5]**, dynamic equations of a PRIP are suggested. Then, these authors successfully design a swing-up controller for moving both pendulums on equilibrium point. But, in that research, the balancing control is not focused and it can be balanced only 10s after getting equilibrium point. In **[6]**, an LQR controller is designed and proved to work well for PRIP in simulation. However, LQR controller is designed due to exactness of mathematical model of system. PID is a very popular method in robotic control. It can be used for systems which has unclear mathematical model or unclear system parameters. But, PID controller has not been designed for PRIP yet. Thence, this is a new approach to control this high-order SIMO system. In this paper, we propose a structure of PD controller for PRIP. Instead of trial-and-error tests, we use genetic algorithm (GA) to find suitable set of control parameters to stabilize well PRIP at equilibrium point.

2. Mathematical Model

Mathematical model of PRIP is shown in Fig. 1. A DC motor is fixed vertically on a base. The axis of motor is fixed to middle of a straight link, called “arm”. On each side of arm is fixed to a encoder. The axis of each encoder is connected to a link, called “pendulum”. Each pendulum can rotate freely at axis of encoder. The rotational angle of pendulum is measured by encoder.

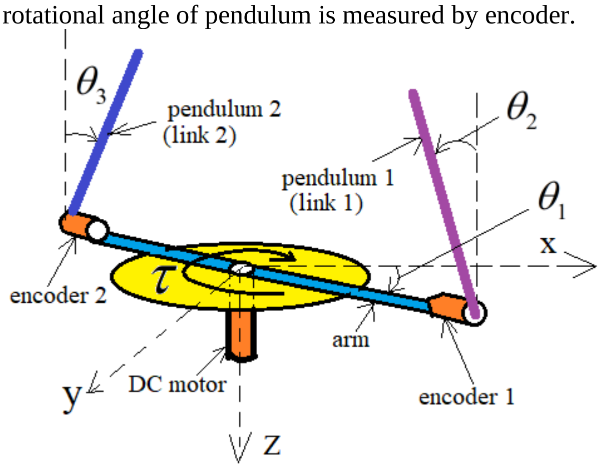

Fig. 1. Mathematical model of PRIP

Parameters and of PRIP is listed in Tab. 1 and Tab. 2.

Tab. 1. System parameters of PRIP

Parameters	Description	Unit
m_1	Mass of arm	kg
m_2	Mass of pendulum 1	kg
m_3	Mass of pendulum 2	kg
l_1	Half of length of arm	m
l_2	Half of length of pendulum 1	m
l_3	Half of length of pendulum 2	m
$J_1 = \frac{m_1 l_1^2}{3}$	Inertial moment of arm	kgm^2
$J_2 = \frac{4 m_2 l_2^2}{3}$	Inertial moment of pendulum 1	kgm^2

$J_3 = \frac{4m_3 l_3^2}{3}$	Inertial moment of pendulum 2	kgm^2
c_1	Friction coefficient of arm	Nmsec/rad
c_2	Friction coefficient of pendulum 1	Nmsec/rad
c_3	Friction coefficient of pendulum 2	Nmsec/rad

Tab. 2. System variables of PRIP

Variables	**Description**	**Unit**
τ	Moment created by DC motor	Nm
θ_1	Angle of arm	rad
θ_2	Angle of pendulum 1	rad
θ_3	Angle of pendulum 2	rad

Kinetic energy of arm is

$$T_1 = \frac{1}{2} J_1 \dot{\theta}_1^2 \tag{1}$$

Kinetic energy of pendulum 1 is

$$T_2 = \frac{1}{2} J_2 \dot{\theta}_2^2 + \frac{1}{2} m_2 \left(\left(\frac{d}{dt} \left(\begin{matrix} l_1 \cos\theta_1 + \\ -l_2 \sin\theta_1 \sin\theta_2 \end{matrix} \right) \right)^2 + \left(\frac{d}{dt} \left(\begin{matrix} l_1 \sin\theta_1 + \\ +l_2 \cos\theta_1 \sin\theta_2 \end{matrix} \right) \right)^2 + \left(\frac{d}{dt} \left(l_2 \cos\theta_2 \right) \right)^2 \right) \tag{2}$$

Kinetic energy of pendulum 2 is

$$T_2 = \frac{1}{2} J_3 \dot{\theta}_3^2 + \frac{1}{2} m_3 \left(\left(\frac{d}{dt} \left(\begin{matrix} l_1 \cos\theta_1 + \\ -l_3 \sin\theta_1 \sin\theta_3 \end{matrix} \right) \right)^2 + \left(\frac{d}{dt} \left(\begin{matrix} l_1 \sin\theta_1 + \\ +l_3 \cos\theta_1 \sin\theta_3 \end{matrix} \right) \right)^2 + \left(\frac{d}{dt} \left(l_3 \cos\theta_3 \right) \right)^2 \right) \tag{3}$$

Kinetic energy of system is

$$T = T_1 + T_2 + T_3 \tag{4}$$

Potential energy of system is

$$V = m_1 g l_1 + m_2 g l_2 \left(1 - \cos\theta_2\right) + m_3 g \left(l_2 - l_3 \cos\theta_3\right) \tag{5}$$

Lagragian operator is defined as

$$L = T - V \tag{6}$$

By Lagrange method, dynamic equations of PRIP are

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \varsigma_i \left(i = 1,2,3\right) \tag{7}$$

where: $q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$; $\varsigma = \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{bmatrix} = \begin{bmatrix} \tau - c_1\dot{\theta}_1 \\ -c_2\dot{\theta}_2 \\ -c_3\dot{\theta}_3 \end{bmatrix}$

Thence, from (7), dynamic equations of PRIP can be written as this form

$$\dot{x} = f\left(x\right) + g\left(x\right)\tau \tag{8}$$

where: $\dot{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T = \begin{bmatrix} \theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2 & \theta_3 & \dot{\theta}_3 \end{bmatrix}^T$;

$f\left(x\right) = \begin{bmatrix} 0 & f_1\left(x\right) & 0 & f_2\left(x\right) & 0 & f_3\left(x\right) \end{bmatrix}^T = \begin{bmatrix} 0 & f_1 & 0 & f_2 & 0 & f_3 \end{bmatrix}^T$;

$g\left(x\right) = \begin{bmatrix} 0 & g_1\left(x\right) & 0 & g_2\left(x\right) & 0 & g_3\left(x\right) \end{bmatrix}^T = \begin{bmatrix} 0 & g_1 & 0 & g_2 & 0 & g_3 \end{bmatrix}^T$

f(x) and g(x) are complicated and they can only described in Matlab code.

The first basic problem of PRIP control is balancing it on equilibrium point which has this condition:

$$x_i = 0\left(i = 1,2,...6\right) \tag{9}$$

At this equilibrium point, the arm is stabilized at position zero when both pendulums are stabilized at up-position without falling down.

3. PID Controller

3.1. PID Controller Design

PID algorithm is very popular in industry and academy. It can be designed without knowing the dynamic equations of system. PID is a SISO structure as in Fig. 2. But, PRIP is a high-order SIMO system. Thence, we proposed a combined structure of PID controllers to stabilize this model as in Fig. 3. In that figure, k1, k2, k3 are gains which have values of -1 or 1. These values have to be chosen through analysis. Parameters of controller j (j=1:3) in Fig. 3 are Kpj, Kij, Kdj. The controller in Fig. 3 can be applied for PRIP if we follow the schematic in Fig. 4.

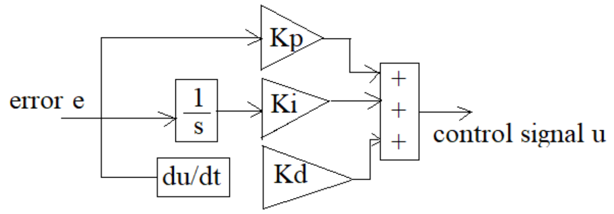

Fig. 2. Basic structure of a PID controller

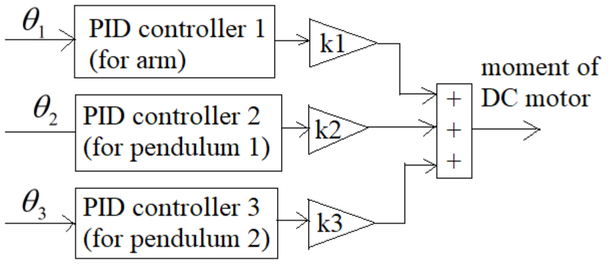

Fig. 3. Structure of a PID controller for PRIP

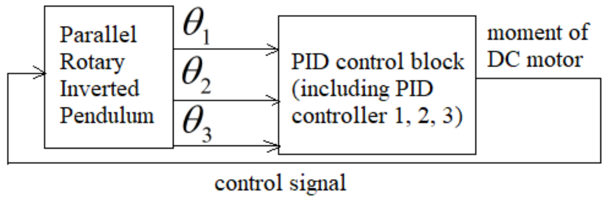

Fig. 4. PID control structure for PRIP

From Fig. 1, we obtain some rules in controlling PRIP as follow:

- If θ_1 is positive, then, τ should be negative to keep arm at position zero. So, k1= -1
- If θ_2 is positive, then, τ should be positive to keep arm at position zero. So, k2= 1
- If θ_3 is positive, then, τ should be positive to keep arm at position zero. So, k3= 1

Thence, values of k1,k2,k3 are

$$k1=-1;\ k2=1;\ k3=1 \tag{10}$$

PID parameters of controllers 1,2,3 in Fig. 3 can be chosen, theoritically, by trial-and-error test. However, this selection is difficult to be done due to big number quantity of parameters which have to be selected. Thence, GA is a solution for this problem.

3.2. GA Program Design

GA imitates the revolution of animals in nature. Then, quality of results is better through generations to satisfy fitness function J which is designed as

$$J = \sum_{i=1}^{10001} \theta_{1i}^2 + \sum_{i=1}^{10001} \theta_{2i}^2 + \sum_{i=1}^{10001} \theta_{3i}^2 \tag{11}$$

where: :

• 10001 is number of samples collected from simulation program. Simulation time is 100 seconds. Sample time is 0.01 second. Thence, number of samples is 10001.

• θ_{ji} is value of θ_j at sample i (i=1:10001; j=1:3)

Which set of control parameters making J smaller is better set of PID controller. Initial value of fitness function Jmin is chosen to be 4000. Fitness function J will be updated with smaller value after each generation.

Initial values of variables in Tab. 2 are selected to be near the equilibrium point (9) as

$$x1=0.01;\ x2=x3=x4=x5=x6=0 \tag{12}$$

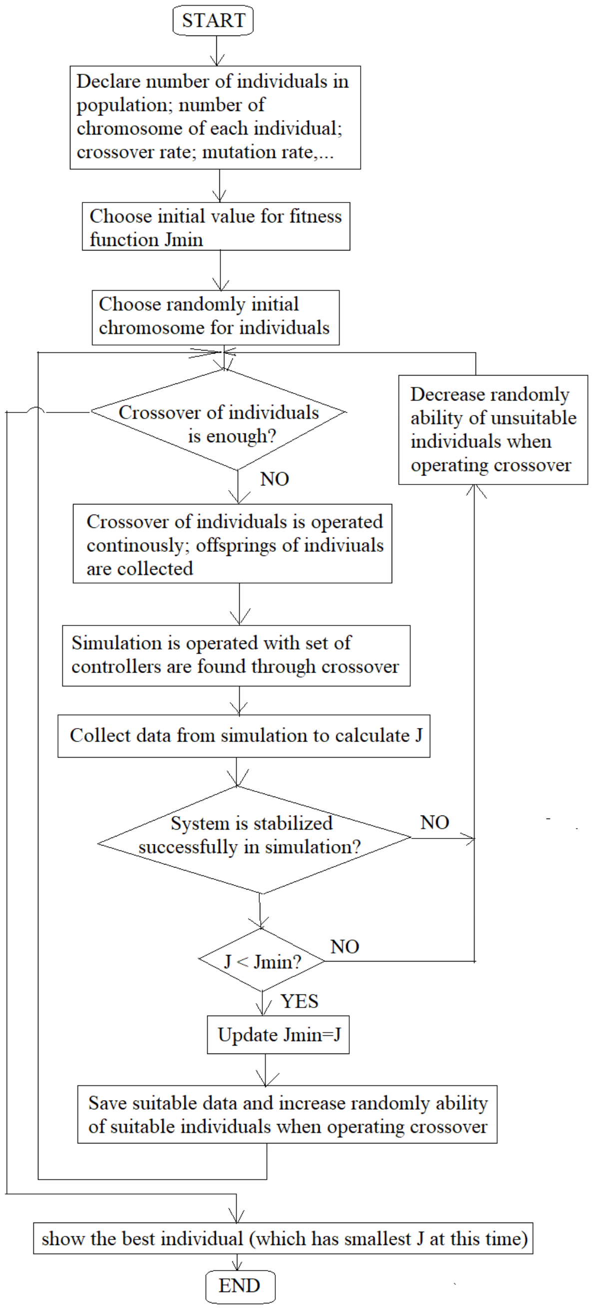

Fig. 5. Diagram of GA program to find and optimize PID parameters

Cossover coefficient is chosen 0.8; Mutation coeficient is chosen 0.2. Population is chosen 2000 individuals.

4. Simulation Results

4.1. PID Control

There are two sets of PID parameters at generation 4500th (set 1) and 10000th (set 2) that can balance well PRIP in Matlab/Simulink simulation. The values of fitness function in each case are 810 and 22 (in Fig. 6).

Set 1 is:

Kp1=3.2562; Ki1=0.0127; Kd1=1.2534; (13)
Kp2=12.4458; Ki2=0.6792; Kd1=9.2654;
Kp3=4.2305; Ki2=5.3378; Kd1=6.2220

Set 2 is

Kp1=0.0344; Ki1=0.4251; Kd1=1.9034; (14)
Kp2=5.3277; Ki2=0.2271; Kd1=8.3296;
Kp3=9.3406; Ki2=7.9044; Kd1=11.5809

The value of fitness function through generation is shown in Fig. 6. This value decreases from 4000 to around 20 when generations move from 1 to 10000. Thence, the quality of response of PRIP under set 2 is better than under set 1. Later generation give better set of PID control.

After generation 10000th, this value does not change much. The quality of PID controller is not improved even though GA still run. This can be due to local extremes. However, in this paper, we neglect this. But, in future research, we can optimize the searching by increase mutation coefficient to overcome local extremes.

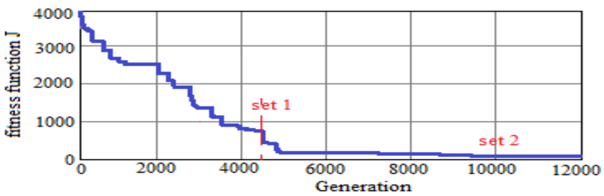

Fig. 6. Value of fitness function through generations

Simulation results of response of system under two sets of controllers in (13) and (14) are shown in Fig. 7, Fig. 8, Fig. 9. In those figures, PID algorithm is proved to control well PRIP after control parameters are found by GA. Settling time in all case is around 4.2 seconds. GA is proved to a suitable solution in finding control parameters for PID algorithm.

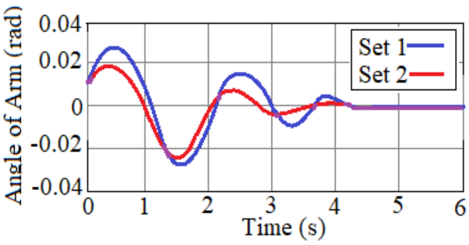

Fig. 7. Angle of arm (rad)

In Fig. 7, oscillation of arm is smaller under set 2 (from –0.023 rad to 0.02 rad) than under set 1 (from -0.022 rad to 0.024 rad).

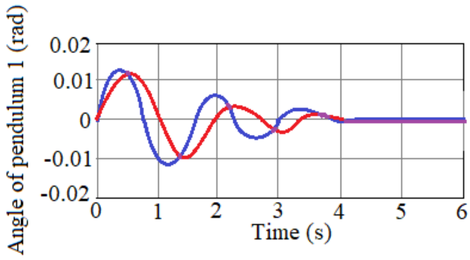

Fig. 8. Angle of pendulum 1 (rad)

In Fig. 8, oscillation of pendulum 1 is smaller under set 1 (from -0.01 rad to 0.011 rad) than under set 2 (from –0.011 rad to 0.012 rad).

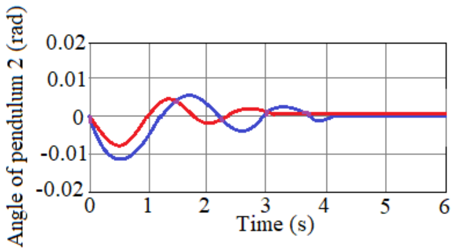

Fig. 9. Angle of pendulum 2 (rad)

In Fig. 9, oscillation of pendulum 2 is smaller under set 1 (from -0.007 rad to 0.005 rad) than under set 1 (from –0.011 rad to 0.006 rad)

From simulation results, the oscillation of all components of PRIP is smaller under set 2 than under set 1. The settling time is approximately the same in all cases. Thence, it suits the fact that controller in set 2 is better than under set 1. Thence, GA is proved to be able to optimize control parameters for PID algorithm.

4.2. PD Controller

There are 9 individuals (Kp1, Ki1, Kd1, Kp2, Ki2, Kd2, Kp3,Ki3, Kd3) in a set of control parameters. Thence, it causes more time in running GA algorithm. From **[6]**, an LQR controller is suitable for balancing PRIP when only 6 control parameters are used (K1, K2,…, K6 in matrix K). Besides, LQR structure is similar to PD structure. Thence, in order to decrease the time of running GA, we propose to let

Ki1=0; Ki2=0; Ki3=0 (15)

Thence, only 6 parameters Kp1, Kd1, Kp2, Kd2, Kp3, Kd3 are found and optimized by GA. By using GA as in Fig. 5, after 20000 generation, we obtain

Kp1=7.2599; Kd1=20.3442; Kp2=9.5835; (16)
Kd2=1.2341; Kp3=9.4553; Kd3=3.3899

The simulation results of controlling PRIP by controller with control parameters as in (15), (16) are described in Fig. 10

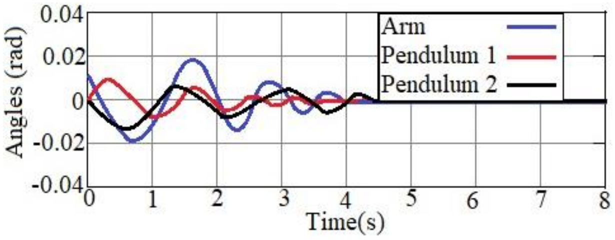

Fig. 10. Angles of arm, pendulums under PD controller

PRIP is stabilized after 4.5 seconds by PD controller. The oscillations of angles are similar to that under PID controller. The generations of searching is 20000 is bigger than 10000 in Fig. 6 but the speed of searching is faster and total time is shorter (as about 6 hours instead of approximate 8 hours). Then, we can optimize GA program by making PIC controller to be simpler by regarding it as a PD controller.

5. Conclusion

In this paper, we propose a PID controller to stabilize PRIP at equilibrium point. GA is used to find and optimize successfully suitable PID control parameters. To shorten the time of GA running, we also suggest simplify PID controller to PD controller. The PD control structure is proved to still work well with similar quality control but with faster GA running.

6. Acknowledgement

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7. References

[1] Link: https://www.quanser.com/products/rotary-inverted-pendulum/

[2] Sarkar T.T., Dewan L., Mahanta C.: "Real Time Swing up and Stabilization of Rotary Inverted Pendulum System", International Conference on Computational Performance Evaluation (ComPE), pp. 517-522, 2020.

[3] Link: https://www.quanser.com/products/rotary-double-inverted-pendulum/

[4] Link: https://www.quanser.com/quanser-community/research-papers/?fwp_research_papers_related_products=1738

[5] Zhang D., Cong S., Li Z., Qin Z.: „The Study of Swing Up and Balance Control for Rotary Parallel Inverted-pendulum", Proceedings of the 4th World Congress on Intelligent Control and Automation, pp. 2370-2374, China, 2002.

[6] Tai V.M. et al.: „A Study of LQR Control for Balancing Rotary Parallel Inverted Pendulum System", Journal of Science of Tien Giang University, 2022 (in progress).