LQR CONTROL FOR DOUBLE-LINKED ROTARY INVERTED PENDULUM: SIMULATION AND EXPERIMENT

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Abstract: Double-linked rotary inverted pendulum is a developed structure of single-linked rotary inverted pendulum. Due to its high-order SIMO structure, it is a big challenge to operate this model on experiment. In this paper, we present an experimental double-linked rotary inverted pendulum that is constructed in our laboratory. A classical linear method – LQR – is used to control well this model in simulation. In experiment, we have stabilized this model for two seconds. Thence, our research contributes experimental results to confirm the ability of LQR for this kind of real model in stabilizing it in equilibrium point.

Keywords: LQR; double-linked; rotary inverted pendulum; SIMO structure.

1. Introduction

Rotary inverted pendulum (RIP) is a classical model for testing control algorithm [1]. It is so popular that Quanser company created a standard platform for this model [2]. When the researches on RIP are adequate, the challenge of control is increased by adding one more link to create double-linked rotary inverted pendulum (DRIP). Quanser company recently presents a DRIP model for laboratory [3]. However, the studies for an experimental DRIP non-Quanser have not presented yet. Thence, in this paper, we present our own experimental DRIP. A linear method which generates a simple and effective control signal is used in this study is LQR. This method is popularly used for these kinds of system [4]. The stability of system under LQR is guaranteed by mathematics [5]. The system parameters and dynamic equations are necessary in designing LQR controller. Therefore, the success in controlling DRIP by LQR methods can prove the effectiveness of mechanical structure of our experimental DRIP. Moreover, we confirm again the ability of LQR methods which was presented before in [6].

2. Dynamic Equations

From **[7]**, when moment is considered as input, dynamic equations of DRIP are listed in (7)-(9) through calculations from (1)-(9).

Potential energy of system is

$$P = m_3 g l_2 \cos\theta_2 + m_3 g\left(L_2 \cos\theta_2 + l_3 \cos\theta_3\right) \quad (1)$$

Kinetic energy of system is

$$\begin{aligned} K &= \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}J_2\dot{\theta}_2^2 + \frac{1}{2}J_3\dot{\theta}_3^2 + \\ &+ \frac{1}{2}m_2\left[\begin{matrix}\left(L_1\dot{\theta}_1 + l_2\dot{\theta}_2\cos\theta_2\right)^2 + \\ +\left(-l_2\dot{\theta}_2\sin\theta_2\right)^2\end{matrix}\right] + \\ &+ \frac{1}{2}m_3\left[\begin{matrix}\begin{pmatrix}L_1\dot{\theta}_1 + l_2\dot{\theta}_2\cos\theta_2 + \\ +l_3\dot{\theta}_3\cos\theta_3\end{pmatrix}^2 + \\ +\left(-l_2\dot{\theta}_2\sin\theta_2 - l_3\dot{\theta}_3\sin\theta_3\right)^2\end{matrix}\right]\end{aligned} \quad (2)$$

where: m_1 (kg), m_2 (kg), m_3 (kg) are mass of arm, link 1 and link 2; L_1(m), l_2(m), l_3(m) are length of arm, link 1, link 2; J_1(kgm^2), J_2(kgm^2), J_3(kgm^2) are inertial moment of arm, link 1, link 2; g(m/s^2) is gravitational acceleration.

Lagragian operator is

$$L = K - P \quad (3)$$

Dynamic equations of DRIP are listed by Euler-Lagrange method are

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}_1} - \frac{\partial L}{\partial\theta_1} = \tau - b_1\dot{\theta}_1 \quad (4)$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}_2} - \frac{\partial L}{\partial\theta_2} = -b_2\dot{\theta}_2 \quad (5)$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}_3} - \frac{\partial L}{\partial\theta_3} = -b_3\dot{\theta}_3 \quad (6)$$

where: b_1, b_2, b_3 is friction parameters of motor and arm, arm and link 1, link 2 and link 2.

Generating (4)-(6), we obtain

$$\tau = h_1\ddot{\theta}_1 + h_2\cos\theta_2\ddot{\theta}_2 + h_3\cos\theta_3\ddot{\theta}_3 + \\ +b_1\dot{\theta}_1 - h_2\dot{\theta}_1\sin\theta_2 - h_3\dot{\theta}_3^2\sin\theta_3 \quad (7)$$

$$0 = -h_2\ddot{\theta}_1\cos\theta_2 - h_4\ddot{\theta}_2 + \\ -h_5\cos(\theta_2-\theta_3)\ddot{\theta}_3 + \\ -b_2\dot{\theta}_2 - h_5\dot{\theta}_3^2\sin(\theta_2-\theta_3) + h_7\sin\theta_2 \quad (8)$$

$$0 = -h_3\ddot{\theta}_1\cos\theta_3 - h_5\cos(\theta_2-\theta_3)\ddot{\theta}_2 + \\ -h_6\ddot{\theta}_3 - b_3\dot{\theta}_3 + h_5\dot{\theta}_2^2\sin(\theta_2-\theta_3) + \\ +h_8\sin\theta_3 \quad (9)$$

where: $h_1 = J_1 + L_1^2(m_2+m_3)$; $h_2 = L_1(m_2l_2 + m_3L_2)$; $h_3 = L_1m_3l_3$; $h_4 = J_2 + L_2^2m_3 + l_2^2m_2$; $h_5 = L_2m_3l_3$; $h_6 = J_3 + l_3^2m_3$; $h_7 = (m_2l_2 + m_3L_2)g$; $h_8 = m_3l_3g$

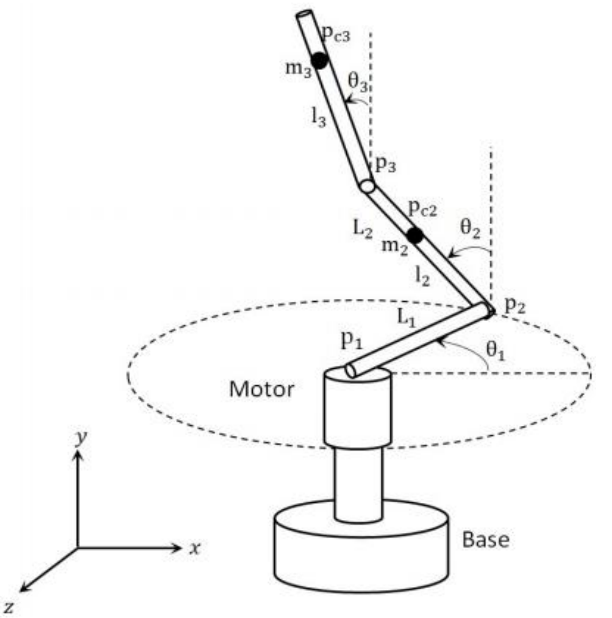

Fig. 1. Mathematical model of DRIP [7]

However, if we focus on the real model, input signal which is moment of motor is not suitable. In that case, voltage on motor is real input control. Thence, a relation between moment τ and voltage u

$$\tau = \frac{K_t u}{R_a} - \frac{K_t K_v \dot{\theta}_1}{R_a} \quad (10)$$

where K_t, K_v, R_a is parameters of DC motor

Re-define the system variables as

$$x_1 = \theta_1;\quad x_2 = \dot{\theta}_1;\quad x_3 = \theta_2;\quad x_4 = \dot{\theta}_2;\quad x_5 = \theta_3;\ x_6 = \dot{\theta}_3 \quad (11)$$

Replace (10) and (11) into (7)-(9), after calculation, we obtain dynamic equations as this form

$$\dot{x}_1 = x_2 \quad (12)$$

$$\dot{x}_2 = f_1(x,u) \quad (13)$$

$$\dot{x}_3 = x_4 \quad (14)$$

$$\dot{x}_4 = f_2(x,u) \quad (15)$$

$$\dot{x}_5 = x_6 \quad (16)$$

$$\dot{x}_6 = f_3(x,u) \quad (17)$$

where $x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T$

The equilibrium point (x_0,u_0) is chosen as

$$x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = 0;\ u = 0 \quad (18)$$

At this equilibrium point, the arm is kept at stable position when the links are kept at up-position. The angle velocities of these components are kept in zero value. Around this point, DRIP can be regarded as linear from as

$$\dot{x} = Ax + Bu \quad (19)$$

where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \frac{\partial f_1}{\partial x_5} & \frac{\partial f_1}{\partial x_6} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} & \frac{\partial f_2}{\partial x_5} & \frac{\partial f_2}{\partial x_6} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} & \frac{\partial f_3}{\partial x_5} & \frac{\partial f_3}{\partial x_6} \end{bmatrix}\Bigg|_{\substack{x = x_0 \\ u = u_0}}$$

$$;\ B = \begin{bmatrix} 0 & \frac{\partial f_1}{\partial u} & 0 & \frac{\partial f_2}{\partial u} & 0 & \frac{\partial f_3}{\partial u} \end{bmatrix}^T\Bigg|_{\substack{x = x_0 \\ u = u_0}}$$

3. Control Algorithm

3.1. General Form

Through [7], DRIP can be stabilized at equilibrium point in (17) if motor is supplied a voltage which is

$$u = -Kx \quad (20)$$

where K is calculated by soving Ricatti equation. But, this process is so complicated that Matlab provides a tool to calculate this matrix K through this command

K = lqr(A,B,Q,R) (21)

In formula (21), K is suitable for a continuous controller. In the case that controller is discrete, K is calculated through another command which is

$$K = dlqr(A_d,B_d,Q,R,T) \tag{22}$$

where:
+ T is sample-time of discrete controller. If it is long, the control signal could be late in controlling system. If it is short, the requirement of a very fast controller appears and price of control board for experiment will increase. Thence, T should be chosen to be suitable. In the case of DRIP, experimental experiences show that it should be 10ms
+ A_d, B_d are discrete forms of A and B. These matrixes are calculated in Matlab as

$$A_d=c2d(A,T) \tag{23}$$
$$B_d=c2d(B,T) \tag{24}$$

+ Matrixes Q and R are weighing matrixes which are selected to get suitable quality.
Forms of Q and R are

$$Q=\begin{bmatrix} Q_1 & 0 & \cdots & 0 \\ 0 & Q_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q_6 \end{bmatrix};\ R=R_1 \tag{25}$$

In (25), $Q_i>0$ (i=1:6) and $R_1>0$. These components can be selected through calibration. The stability of system under LQR controller is guaranteed by mathematics. However, system is stabilized "around" the equilibrium point. And, no limitation of the range around equilibrium point is defined. Thence, if system variables are far from working point, system will fall down. To find the optimal matrixes Q and R and have the suitable K from (22), genetic algorithm (GA) is used. This searching method is popular in optimizing control parameters [8]. The better set of control parameters gives the better simulation and experimental results. Then, this better set can optimize the success of controller in stabilizing system.

3.2. Method in Detail

System parameters for both simulation and experiment are measured and listed in (26)

$$K_b \approx K_v = 0.064943\,(\text{Vs/rad});\ R_a = 6.835271\,(\Omega) \tag{26}$$

Other system parameters from (7)-(9)

m_1=0.031(kg); m_2=0.025(kg); m_3=0.025 (kg); L_1=0.16(m); l_2=0.23(m); l_3=0.23(m); J_1=0.0019(kgm^2); J_2=0.000466(kgm^2); J_3=0.000466(kgm^2); g=9.81(m/s^2) (27)

Using the dynamic equations in (12) to (17), system parameters in (26), matrixes A and B become

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 149.7 & -0.17 & -95.57 & 0 & 8.63 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -356.14 & 0.42 & 328.1 & 0 & -53.61 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 283.45 & -0.33 & -418.41 & 0 & 160.91 & 0 \end{bmatrix}; \tag{28}$$

$$B=\begin{bmatrix} 0 & 2.6870 & 0 & -6.3925 & 0 & 5.0878 \end{bmatrix}^T$$

After changing form of A and B from continuous to discrete, we obtain

$$A_d=\begin{bmatrix} 1.01 & 0.01 & -0.00 & -0.00 & 0.00 & 0.00 \\ 1.51 & 1.01 & -0.96 & -0.00 & 0.09 & 0.00 \\ -0.02 & -0.00 & 1.02 & 0.01 & -0.00 & -0.00 \\ -3.59 & -0.01 & 3.30 & 1.02 & -0.54 & -0.00 \\ 0.01 & 0.00 & -0.02 & -0.00 & 1.00 & 0.01 \\ 2.87 & 0.01 & -4.22 & -0.02 & 1.62 & 1.00 \end{bmatrix}; \tag{29}$$

$$B_d=\begin{bmatrix} 0 & 0.03 & 0 & -0.06 & 0 & 0.05 \end{bmatrix}^T$$

Using GA, after 2000 generations, matrixes Q and R are found as

$$Q=\begin{bmatrix} 1.34 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.67 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6.89 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.52 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10.56 \end{bmatrix}; \tag{30}$$

$$R=0.56$$

From (22), we obtain

$$K=[111.4665 \quad 41.9074 \quad -133.9893 \quad \ldots \quad 32.3769 \quad 323.4086 \quad 33.3219] \tag{31}$$

4. Simulation

We choose initial values of variables of system as

$$x\_init=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.01 \end{bmatrix}^T \tag{32}$$

Then, system under controller from (20) and (31) has the responses which are shown in Fig. 2.

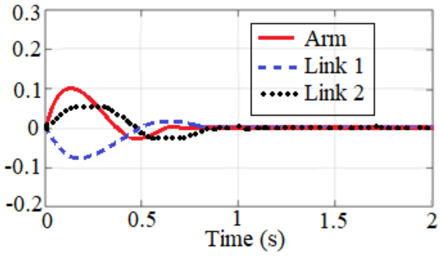

Fig. 2. Angles (rad) of arm and links

In Fig. 2, system is stabilized successfully after 1s. The arm oscillates 0.1 rad around working position before being stable. Link 1 and 2 oscillate 0.08 rad and 0.06 rad, respectively, around the up-right position. Thence, through simulation, LQR controller is confirmed again to control well system

5. Experiment

5.1. Experimental Model

Real DRIP was made in control system laboratory in Ho Chi Minh city University of Technology and Education (HCMUTE). Its components are shown in Fig. 3. This model is developed from RIP in **[9]**.

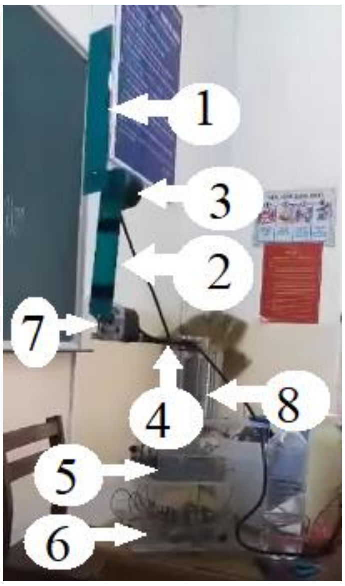

Fig. 3. Experimental RIP in TOP position

1- Link 2
2- Link 1
3- Encoder 2 which measures angle of link 2
4- Arm
5- DC source
6- Control board and driver circuits
7- Encoder 1 which measures angle of link 1
8- DC motor that controls arm

Control board is STM32F4, which can be downloaded by Matlab program. Two incremental encoders are used to measures angles of link 1 and link 2. Angle of arm is measure by another encoder that is fixed to axis of DC motor. Whole model is created of MICA material.

5.2. Experimental Results

Control video clip of our project is shown in [10]. Under controller which is presented in simulation, responses of system are shown from Fig. 4 to Fig. 6. System is stabilized in around 2s. In Fig. 4, the arm rotates around 30^0 before becoming to 40 degree in second 2. In Fig. 5, link 1 is stabilized well in 2 seconds. However, from second 1.9, the angle of link 1 becomes bigger. Fig. 6, after 2s, link 2 oscillates more than 20^0 and falls down. It makes all other components unbalanced.

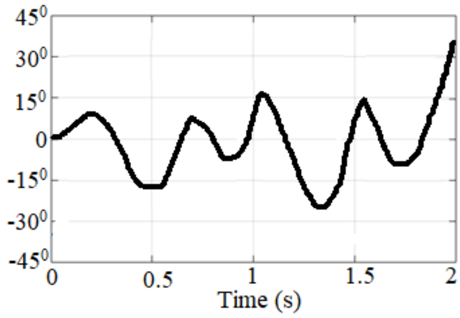

Fig. 4. Angle of arm (degree)

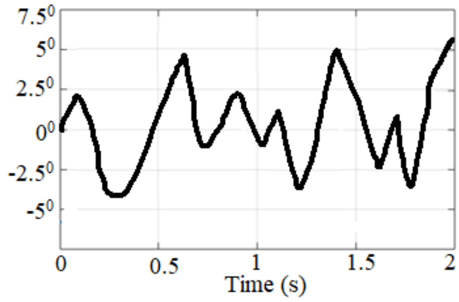

Fig. 5. Angle of link 1 (degree)

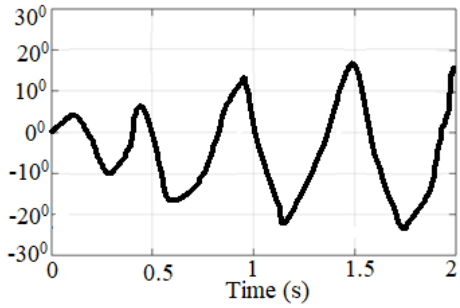

Fig. 6. Angle of link 2 (degree)

Through experimental results, the LQR method needs 2 seconds to keep DRIP not falling. After second 2, the motion of link 2 can not be kept well and it makes all both links fall down. Due to sensitivity of link 2, when value is over 5^0 is the sign of the fall (Fig. 6). From our experience in choosing suitable control parameters, we obtain that if the optimization of GA is not used, the DRIP falls down faster than 2s and if Q and R are chosen to be unit matrixes, our DRIP can not be balanced even within 0.5s.

6. Conclusions

In our research, we present a DRIP that is controlled well by LQR method. Although DRIP model has been controlled well by LQR method through standard Quanser platform, we confirm the method through a model which is made in laboratory. The simulation proves the ability of this classical method to give the basement for the experiment. In experiment, our DRIP can be balanced in 2 seconds before falling down.

Due to GA, acceptable control parameters can give that result. If only standard without calibration, successful LQR controller in simulation can still not work well in experiment. Thence, the exactness of our model still needs to be improved in mechanical structure. However, a GA-LQR algorithm, in this case, can be applied acceptably for experimental DRIP.

7. References

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