LQR-BASED INFORMATION FUSION FUZZY CONTROLLER FOR EXPERIMENTAL DOUBLE INVERTED PENDULUM ON CART

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Abstract: This study presents the experimental implementation of an LQR-based Information Fusion Fuzzy Logic Controller (IF-FLC) for the double inverted pendulum on a cart (DIPC), extending previous LQR and ANFIS investigations. The proposed controller integrates information fusion to condense multi-state measurements into compact fuzzy indicators, enabling a reduced-rule architecture while preserving essential system dynamics. Through both simulation and real-time experiments, the IF-FLC demonstrates notable improvements in cart stabilization accuracy and steady-state damping compared with conventional LQR and LQR-based ANFIS controllers. While LQR achieves the smallest angular deviations, the IF-FLC significantly lowers cart-position RMSE and reduces control voltage, resulting in smoother, quieter, and more energy-efficient operation. Experimental results confirm that the IF-FLC offers a superior balance between regulation precision and control effort, highlighting its effectiveness as a practical intelligent control solution for nonlinear, underactuated systems such as the DIPC.
Keywords: Double Inverted Pendulum, Single Input – Multi Output System, Fuzzy Logic Control, Information Fusion, Intelligent Control.

1. Introduction

Double-linked inverted pendulum on cart (DIPC) is widely regarded as a representative benchmark in nonlinear control due to its strong coupling, underactuation, and intrinsic instability [1, 2]. Owing to these characteristics, it has been extensively used to evaluate advanced control methodologies, particularly optimal linear strategies such as the Linear Quadratic Regulator (LQR) [3]–[8]. While LQR provides effective stabilization near the equilibrium point, its performance may degrade under nonlinear effects and practical uncertainties, motivating development of intelligent control approaches.

Fuzzy logic control has been recognized for its capability to handle nonlinear dynamics without relying strictly on precise mathematical models [9]–[11]. More recently, data-driven neuro-fuzzy systems such as ANFIS have been applied to the DIPC system, demonstrating improved adaptability while preserving stabilization characteristics of LQR-based design [13], [14]. In parallel, information fusion strategies have been introduced to reduce input dimensionality and construct compact fuzzy architectures without sacrificing essential system behavior [12, 15].

In our previous works, DIPC dynamic model was established and validated, followed by the successful implementation of an experimental LQR controller [14] and an LQR-based ANFIS controller [13]. Furthermore, a simulation study of an Information Fusion Fuzzy Logic Controller (IF-FLC) confirmed its potential in enhancing stabilization performance while maintaining a reduced rule base [15].

Building upon these foundations, the present study aims to experimentally evaluate and compare three control strategies LQR [14], ANFIS [13], and the proposed IF-FLC under both simulation and real-time conditions. The objective is to highlight the practical advantages of the IF-FLC approach in terms of stabilization accuracy, robustness, and control efficiency for the DIPC system.

2. DIPC Model

A detailed derivation of the nonlinear dynamic model of the double inverted pendulum on cart (DIPC) has been established and experimentally validated in our previous studies [12, 13]. In the present work, this established model is directly employed as the basis for controller design and performance evaluation. To ensure continuity, only a concise overview of the model structure is briefly recalled in the following section, while the complete formulation and validation procedures are available in [12, 13].

The physical configuration of the DIPC system and the associated reference coordinate frames are defined in Fig. 1.

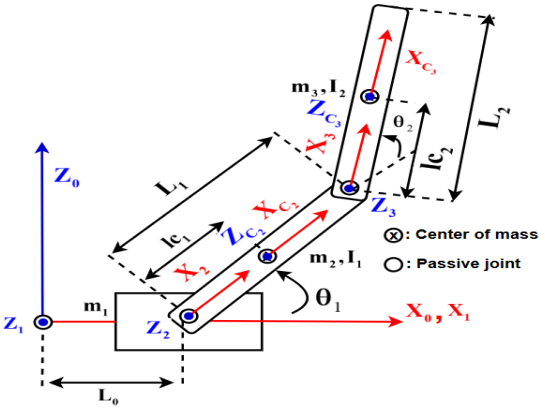


Fig. 1. Structure of the system [15]

3. Information Fusion Fuzzy Controller

3.1. LQR-Based Foundation

In our previous works [12, 14], an LQR-based control framework for the DIPC system was developed, where the optimal gain matrix K was derived and later used to design the IF-FLC controller. In this study, the same LQR structure is experimentally implemented, with the last gain adjusted from 24.7514 to 14.7514 for practical operation. The modified matrix serves as the reference for the proposed IF-FLC controller. The LQR structure is shown in Fig. 2.

$$K = \begin{bmatrix} K_{L_0} & K_{\dot{L}_0} & K_{\theta_1} & K_{\dot{\theta}_1} & K_{\theta_2} & K_{\dot{\theta}_2} \end{bmatrix} \quad (1)$$

Where:

$K_L = 0.9763, K_{\dot{L}} = 4.9193, K_{\theta_1} = -30.8268$

$K_{\dot{\theta}_1} = -15.2867, K_{\theta_2} = -136.3463, K_{\dot{\theta}_2} = -14.7514$

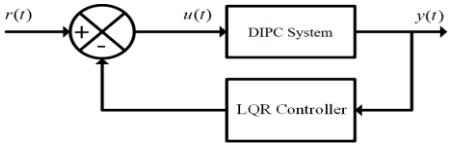


Fig. 2. LQR controller structure of DIPC system [14]

3.2. Design of the Information Fusion FLC

As the detailed formulation is provided in [15], this section briefly outlines the key steps in designing the IF-FLC controller.

$$F(x) = \begin{bmatrix} \frac{K_L}{K_{\theta_2}} & \frac{K_{\theta_1}}{K_{\theta_2}} & \frac{K_{\theta_2}}{K_{\theta_2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{K_{\dot{L}}}{K_{\dot{\theta}_2}} & \frac{K_{\dot{\theta}_1}}{K_{\dot{\theta}_2}} & \frac{K_{\dot{\theta}_2}}{K_{\dot{\theta}_2}} \end{bmatrix} \quad (2)$$

At this stage, the six original state variables are condensed into two representative indicators: the overall error (E) and its rate of change (EC).

$$\begin{bmatrix} E \\ EC \end{bmatrix} = F(x)x \quad (3)$$

Where:

$$E = \frac{K_L}{K_{\theta_2}} L_0 + \frac{K_{\theta_1}}{K_{\theta_2}} \theta_1 + \theta_2 \quad (4)$$

$$EC = \frac{K_{\dot{L}}}{K_{\dot{\theta}_2}} \dot{L}_0 + \frac{K_{\dot{\theta}_1}}{K_{\dot{\theta}_2}} \dot{\theta}_1 + \dot{\theta}_2 \quad (5)$$

The structure of the proposed IF-FLC controller is illustrated in Fig. 3.

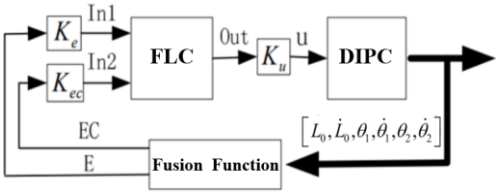


Fig. 3. IF-FLC control structure [15]

Fig. 4 presents the membership functions assigned to the input variables E, EC, and the controller output. Each input and output variable is partitioned into seven triangular membership functions, defined as:

+ NS (Negative Small).
+ NM (Negative Medium).
+ NB (Negative Big).
+ ZE (Zero).
+ PS (Positive Small).
+ PM (Positive Medium).
+ PB (Positive Big)

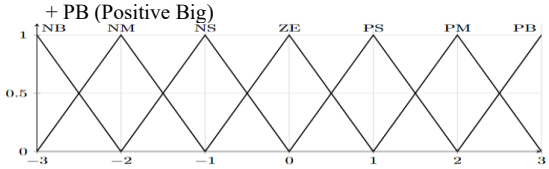


Fig. 4. Membership functions of inputs and output [15]

According to [11], the rules listed in Table 1 are derived from expert heuristics combined with empirical experience.

Table 1. IF-FLC control rules table [11]

E \ EC	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NM	NM	NS	ZE
NM	NB	NB	NM	NM	NS	ZE	PS
NS	NB	NM	NM	ND	ZE	PS	PM
ZE	NM	NM	NS	ZE	PS	PM	PM
PS	NM	NS	ZE	PS	PM	PM	PB
PM	NS	ZE	PS	PM	PM	PB	PB
PB	ZE	PS	PM	PM	PB	PB	PB

4. Simulation Results

In [15], simulation results compared the IF-FLC and LQR controllers. This study extends the comparison to include the LQR-based ANFIS controller [16], providing a comprehensive evaluation of all three strategies for the DIPC system. Simulations were conducted in MATLAB/Simulink (2020a) with a 0.005s sampling time, and initial angles of 0.1 rad (Link 1) and 0.15 rad (Link 2) as illustrated in Fig. 5.

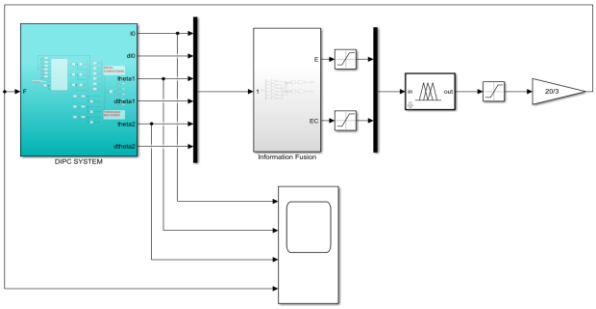


Fig. 5. Simulation of DIPC using IF-FLC controller [15]

As shown in Fig. 6, large initial angular deviations require strong corrective action, leading to transient cart oscillations. These oscillations gradually decay, and the cart motion converges within approximately ±0.2 m, which is acceptable for practical operation.

Compared with the conventional LQR controller, both ANFIS and IF-FLC reduce cart overshoot (about 0.8 m versus 0.95 m). Notably, the IF-FLC exhibits better damping near equilibrium, limiting residual oscillations to around 0.03 m, significantly smaller than the 0.15 m observed under LQR, thereby improving both transient and steady-state performance.

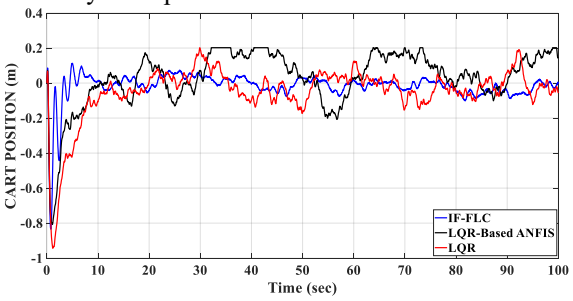


Fig. 6. The cart position response

The angular responses of both pendulums (Fig. 7 and Fig. 8) confirm stable confinement around the upright position under all controllers.

For Link 1, LQR and ANFIS limit the motion to about (−0.05, 0.04) rad, while IF-FLC further reduces it to approximately (−0.03, 0.03) rad. A similar pattern is observed for Link 2: LQR maintains the angle within (−0.02, 0.02) rad, whereas IF-FLC tightens it to around (−0.01, 0.01) rad. Although IF-FLC shows slightly stronger initial transients, it provides better steady-state damping, resulting in smaller residual oscillations near equilibrium.

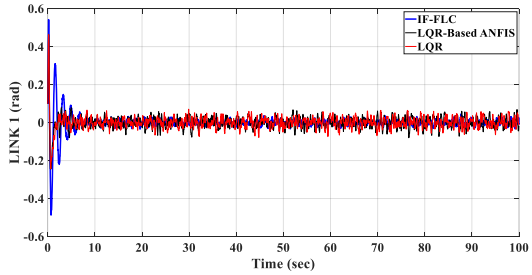


Fig. 7. The first pendulum's response

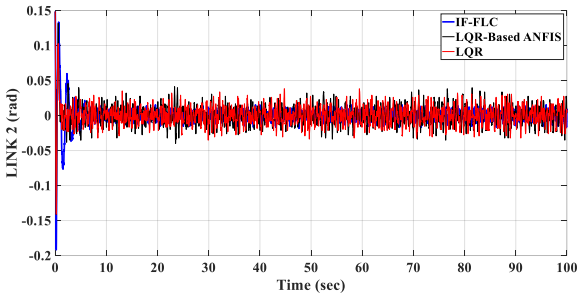


Fig. 8. The second pendulum's response

To assess actuation effort, the control force is converted into an equivalent torque using the pulley pitch radius. As shown in Fig. 9, the torque remains bounded within approximately ±0.13–0.15 N·m.

The IF-FLC requires higher control effort during the initial stabilization phase, leading to slightly larger transient oscillations. However, this stronger action enables faster damping and improved steady-state performance compared to LQR and ANFIS. Overall, the torque levels stay within practical limits, supporting appropriate actuator selection and reliable real-world operation.

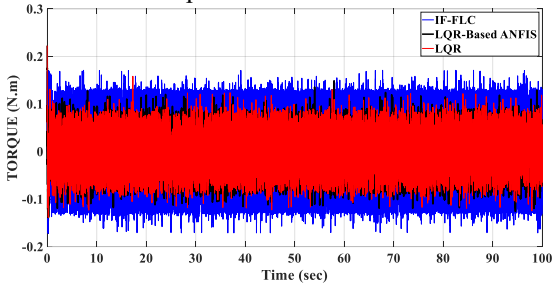


Fig. 9. Applied torque on the cart

Based on Table 2, RMSE results reveal notable differences in the control performance of the DIPC system under LQR, IF-FLC, and ANFIS controllers. For cart motion, IF-FLC achieves the lowest RMSE value (0.0816), significantly outperforming both LQR (0.1608) and ANFIS (0.1614), which indicates improved tracking accuracy and more effective oscillation suppression. Meanwhile, the RMSE values of the angular responses of Link 1 and Link 2 are relatively close for all three controllers, confirming their ability to reliably stabilize the pendulum links. Slightly higher RMSE observed for IF-FLC in the angular states and torque can be mainly attributed to its stronger control action during the transient phase. However, from a long-term perspective, IF-FLC demonstrates a clear advantage in terms of cart positioning and steady-state stability, maintaining small oscillation amplitudes of both pendulums, whereas ANFIS exhibits angular regulation performance comparable to that of LQR controller.

Table 2. Quality of DIPC according to the Root Mean Square Error Standard (Simulation)

Response of system	Controller		
	LQR	IF-FLC	ANFIS
Cart (m)	0.1608	0.0816	0.1614
Link1 (rad)	0.0362	0.0468	0.0363
Link2 (rad)	0.0145	0.0130	0.0154
Torque (Nm)	0.0378	0.0777	0.0379

5. Experiment Results

The hardware setup employed in this study is illustrated in Fig. 10 and Fig. 11 as follows:

1. Motor's encoder.
2. Link 1 and its encoder.
3. Link 2 and its encoder
4. NISCA DC motor.
5. Pulley transmission mechanism (gear ratio 3:1).
6. STM32F407G-DISC1.
7. BTS7960 H-Bridge circuit.
8. Power circuit breaker.
9. Encoder signal reading and UART circuit.

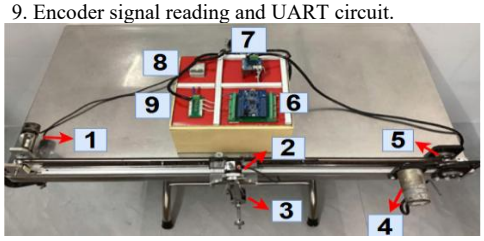


Fig. 10. Hardware setup of the DIPC system

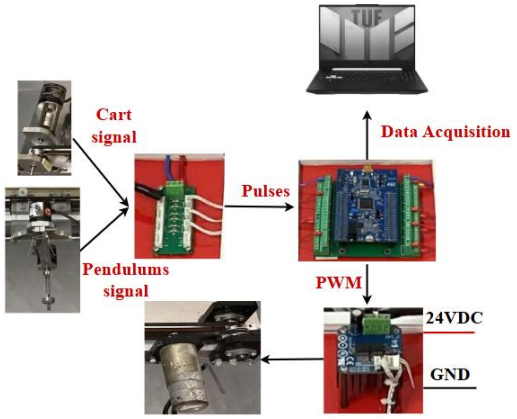


Fig. 11. Connection diagram

Fig. 12 illustrates the processing flow of the DIPC system control program.

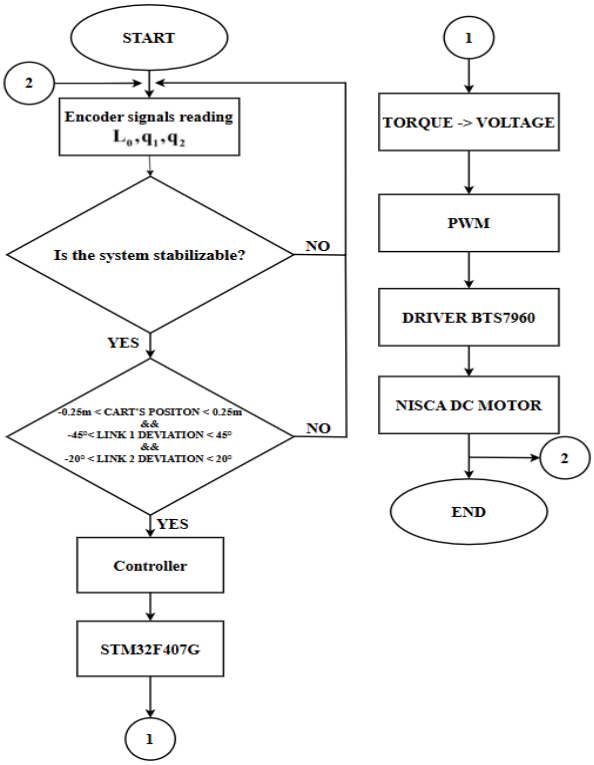


Fig. 12. Algorithm flowchart for DIPC system

Fig. 13 illustrates the experimental cart position responses under the LQR, IF-FLC, and LQR-based ANFIS controllers. All three strategies successfully confine the cart motion within a narrow interval of approximately −0.05 to 0.05 m, confirming stable real-time operation. Compared with the simulation results, the experimental responses show improved behavior, as no noticeable overshoot around the equilibrium position is observed.

Among the tested controllers, the IF-FLC maintains the cart position within the tightest band, mostly around ±0.03 m, indicating superior regulation capability. The ANFIS controller also demonstrates enhanced stabilization compared with the conventional LQR, returning the cart to equilibrium more smoothly and sustaining a slightly more consistent steady-state response. Overall, while all controllers achieve satisfactory stabilization, the intelligent approaches (IF-FLC and ANFIS) provide improved precision in experimental cart positioning relative to the standard LQR.

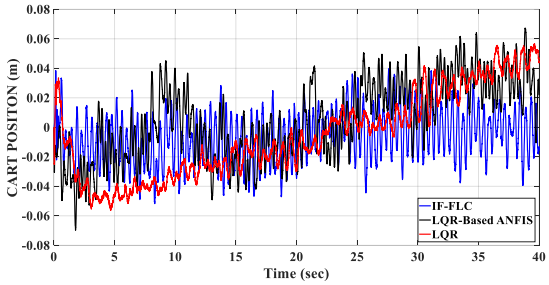


Fig. 13. Experimental response of cart position

Fig. 14 and Fig. 15 illustrate the experimental angular deviation responses of the first and second pendulums under the LQR, IF-FLC, and LQR-based ANFIS controllers. The results indicate that, in all cases, the angular oscillations are confined within approximately ±0.05 rad (≈2.86°), confirming stable upright balance during real-time operation.

While both IF-FLC and ANFIS exhibit slightly larger oscillation amplitudes than the conventional LQR controller, the deviations remain small and practically negligible in experimental conditions. The ANFIS controller, trained from LQR data, successfully reproduces the stabilization behavior, whereas the IF-FLC maintains comparable balance performance despite its stronger nonlinear control action. Overall, the observed angular deviations are well within acceptable limits, ensuring smooth and reliable operation of the DIPC system in practice.

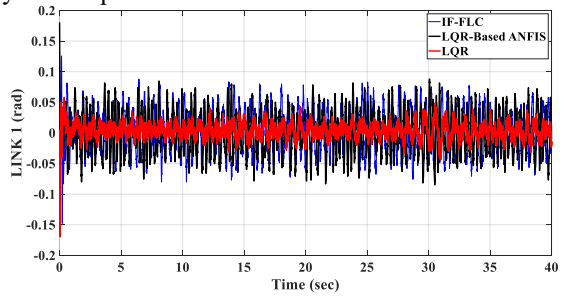


Fig. 14. Experimental response of first pendulum angle

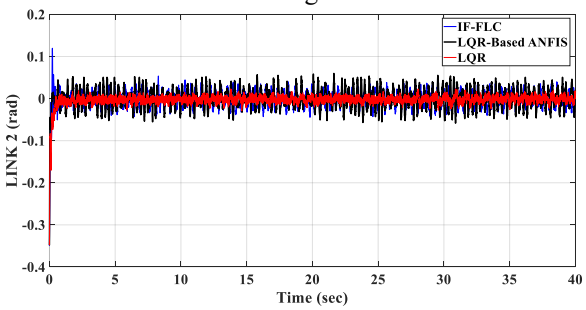


Fig. 15. Experimental response of second pendulum angle

Fig. 16 present the experimental control voltage generated by the LQR, IF-FLC, and LQR-based ANFIS controllers. It can be clearly observed that the LQR controller produces the highest voltage amplitude among the three strategies, which explains the more noticeable mechanical noise during practical operation.

In contrast, both IF-FLC and ANFIS operate with lower voltage levels, resulting in smoother actuator behavior. The ANFIS voltage response appears significantly smaller when plotted on the same scale as LQR, indicating reduced control effort. However, since ANFIS is trained using data generated by the LQR controller, its smoothness remains slightly inferior to that achieved by IF-FLC. Overall, the intelligent controllers demonstrate more efficient voltage utilization and contribute to quieter and more stable system performance compared with the conventional LQR approach.

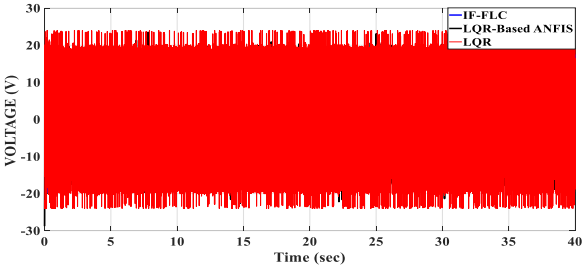


Fig. 16. Actual voltage supplied to the motor

Based on Table 3, clear differences can be observed among the LQR, IF-FLC, and ANFIS controllers in the experimental evaluation. For cart positioning, IF-FLC achieves the smallest RMSE (0.0188 m), outperforming both ANFIS (0.0267 m) and LQR (0.0299 m), which confirms its superior regulation capability around the equilibrium point. The ANFIS controller also improves cart stabilization compared to LQR, though its performance remains slightly inferior to IF-FLC.

In terms of angular stabilization, LQR provides the lowest RMSE values for both Link 1 and Link 2, indicating strong precision in maintaining upright balance. Although IF-FLC and ANFIS exhibit somewhat larger angular errors, the deviations remain within acceptable limits and do not affect overall system stability.

Regarding control effort, both IF-FLC and ANFIS significantly reduce the voltage RMSE compared to LQR, demonstrating more efficient actuator utilization and smoother operation. Overall, IF-FLC offers the best balance between cart positioning accuracy and reduced control energy.

Table 3. Quality of DIPC according to the Root Mean Square Error Standard (Experiment)

Response of system	Controller		
	LQR	IF-FLC	ANFIS
Cart (m)	0.299	0.0188	0.0267
Link1 (rad)	0.0144	0.0366	0.036
Link2 (rad)	0.0135	0.0204	0.0246
Voltage (V)	17.0216	13.6873	13.6137

6. Conclusions

In this study, the IF-FLC controller was successfully implemented on the experimental DIPC system and demonstrated superior overall performance compared with both LQR and LQR-based ANFIS. The experimental results show that IF-FLC achieves the best cart positioning accuracy while maintaining stable pendulum angles within acceptable limits. Moreover, it requires lower control voltage than LQR, resulting in smoother and more energy-efficient operation. These findings confirm that the information fusion strategy enhances practical control performance and provides a more balanced trade-off between stabilization accuracy and control effort, making IF-FLC a promising approach for real-time DIPC applications.

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