# SLIDING MODE CONTROL FOR BALL IN TUBE

**Quang-Thien Nguyen, Huynh-Phuc Le, Quoc-Kiet Dinh, Dieu-Minh-Thu Tran, Thanh-Hung Ngo, Le-Duc-Anh Nguyen, Hong-An Tran, Thi-Thanh-Hoang Le ***

Ho Chi Minh City University of Technology and Education (HCMUTE)
Vo Van Ngan Str., No. 01, Ho Chi Minh City (HCMC), Vietnam
***** Corresponding author. E-mail: hoangltt@hcmute.edu.vn

**Abstract:** This paper presents simulation of Sliding Mode Control (SMC) algorithm and the PID algorithm applied to the ball in tube – a single input single output (SISO) nonlinear system. The primary objective of this study is to stabilize the position of the ball at a desired location inside the tube despite disturbances or variations in initial conditions. Firstly, we focus on designing PID controller, the PID parameters are tuned using standard methods to ensure acceptable transient and steady – state performance. Thence, we proceed to design the Sliding Mode Controller and optimize it by using genetic algorithm (GA) to determine the optimal parameters for the controller, so that the system output can accurately track the desired setpoint. Through a series of simulations are carried out in Matlab/Simulink toevaluate the effectiveness of both controllers. Performance metrics such as rise time, settling time, overshoot, and steady-state error are used for comparison. The simulation results demonstrate that the GA-optimized SMC significantly outperforms the conventional PID controller in terms of tracking accuracy, robustness to disturbances, and control efficiency. These findings highlight the potential of intelligent optimization techniques like GA in enhancing the performance of advanced control algorithms such as SMC in nonlinear dynamic systems.
**Keywords:** PID Control; sliding mode control; ball in tube; SISO system.

## 1. Introduction

In control engineering, SISO systems are a class of systems which are popularly used in both academy and industry, such as, heating oven [1], DC motor [2]... Controlling position of an object in space is also a kind of this class. Magnetic Levitation [3] is a typical system in this case. However, the difficulty of creating this model in reality leads the researchers to focus on similar object – ball in tube system. The effective of magnetic is replaced by rotational speed of a fan to control position of a ball in a tube, by air. Through this equivalent model, PID control and its developed hybrid control – PID-fuzzy control - are proved to be effective in both simulation and experiment [4]. PID method is easy to be designed and it can be optimezed by GA [5]. But, it is not guaranteed by mathematics. The intelligent control, such as neural control or fuzzy control, can make quality of controllers better but they still can not be guaranteed by mathematics. To overcome the mathematical problem, nonlinear control is a solution [6]. By satisfying Lyapunov criteria, the stability of system is guaranteed. Among nonlinear algorithms, SMC is the most popular method due to its easiness in designing Lyapunov function. Besides, SMC method is popularly used in trajectory control for system due to its designing in sliding surfaces. Thence, through this reseach, we want to present our designed SMC controller which is optimized by GA. And, its provebility over classical PID method is shown through simulation. SMC controller is proven to be better than PID control when controlling ball following expeted pulse trajectory. This can be reference for more nonlinear reseaches in the future.

## 2. System Model

In several previous studies, concisely [**7**], ball in tube is illustrated in Fig. 1.

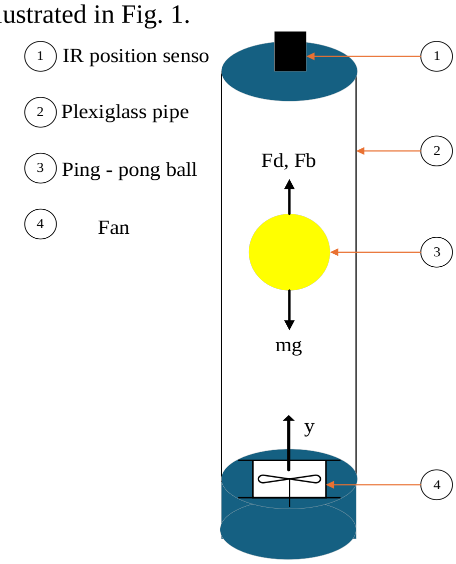


**Fig. 1.** The ball in tube.

The nonlinear description of the air stream, which subjects to the Bernoulli's equation. The system dynamic equations:

$$p_1 + \frac{1}{2}\rho {v_1}^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho {v_2}^2 + \rho g y_2 \qquad (1)$$

Where, $\frac{1}{2}\rho v^2$ is kinetic energy and $\rho g y$ is gravitational potential, $p_1$ and $p_2$ are the static pressures of air at the cross-section, $\rho$ is the density of the following air, $y_1$ and $y_2$ are the different distances between the ball and the bottom of the pipe, $v_1$ and $v_2$ are the mean velocities of fluid flow at the cross-section .

The dynamic equation for the air levitation ball and pipe system from Newton's second law:

$$m\frac{d^2y}{dt^2} = F_b + F_d - F_g \quad (2)$$

$$F_b = \rho g V_b;\ F_d = f(\Delta\rho, F_f);\ F_g = mg \quad (3)$$

Where, $\Delta\rho$ is the air pressure difference, $\rho$ is the density of air, $g$ is the gravitational acceleration, $m$ is the mass of the ball, $V_b$ is the ball's volume and $F_f$ is the friction force caused by airflow.

Based on the above physical and mathematical foundations, the mathematical model of ball in tube system is described as follows [7]:

$$\frac{y(s)}{u(s)} = \frac{b.kv}{s(s+b)(\tau s+1)} \quad (4)$$

Where, $kv$ is the sensitivity gain that relates the input voltage to the wind speed at steady state, and τ is the time constant of the fan.

In the present paper, the effect of the two delays is assumed to be $T_d$ , cumulatively.

$$e^{-T_d s} \approx \frac{1}{T_d s+1} \quad (5)$$

Finally, the transfer function of the entire system defines as follows:

$$G(s) = \frac{y(s)}{u(s)} = \frac{b.kv}{s(s+b)(\tau s+1)(T_d s+1)} \quad (6)$$

The system parameters in
( 6) are selected in Tab. 1

**Tab. 2.** Simulation parameters ball in tube [7]

| $kv$ | $b$ | $\tau$ | $T_d$ |
|---|---|---|---|
| 1 | 1 | 0.025 | 0.01(s) |

### 3. PID Control Solution

The control of the ball in tube system involves two basic requirements. First, it is necessary to stabilize the ball at a desired predetermined position. Second, it is required to control the ball to follow a trajectory. The automatic control system structure for the ball in tube system is constructed as follows:

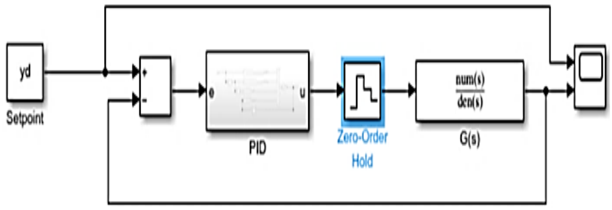


**Fig. 2.** Block diagram of the PID control system for the ball in tube system.

Where, Setpoint (yd) is the desired ball height value (for the trajectory control case, yd will continuously change according to the user-adjusted value), G(s) is the transfer function of the ball in tube

The PID controller is designed as follows:

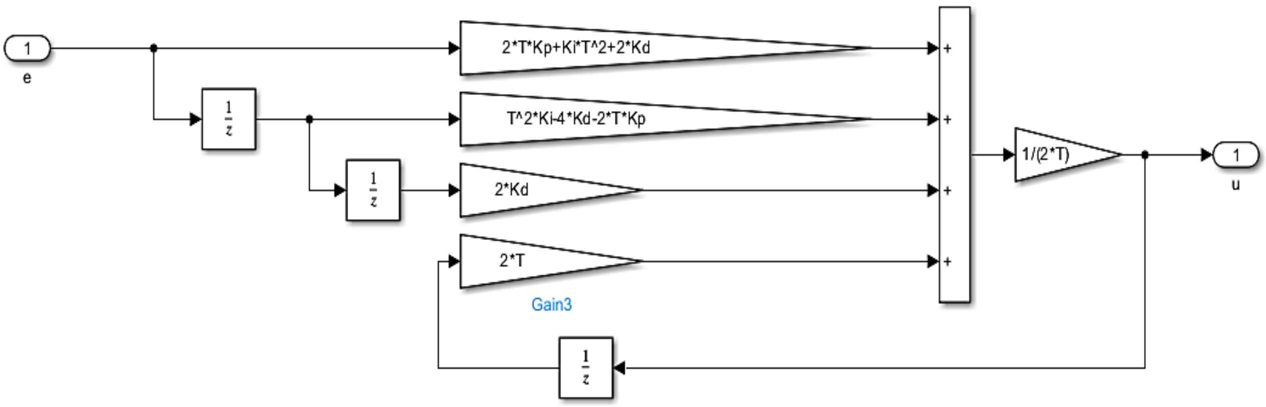


**Fig. 3.** The PID controller.

The block G(s) is constructed based on (6):

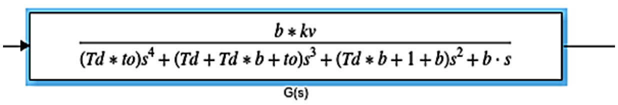


**Fig. 4.** Transfer function representation of the ball in tube system.

### 4. Sliding Mode Control Solution

#### 4.1. Designing a Sliding Mode Control

To build a controller for the ball in tube system:

Step 1: we must first represent system (6) in the form of a differential equation.

The equation (6) is converted to:

$$y(s)[T_d.\tau.s^4+(T_d+T_d.b+\tau)s^3+(T_d.b+1+b)s^2+b.s]=kv.b.u(s) \quad (7)$$

Set the following variables:

$$a=T_d\tau \;;\; c=T_d+T_db+\tau \;;\; d=T_db+1+b \quad (8)$$

The equation (7) is rewritten as follows:

$$y(s)[a.s^4+cs^3+ds^2+b.s]=kv.b.u(s) \quad (9)$$

Apply Laplace transform to equation (7), we have the following differential equation:

$$\dddot{y}(t)a+\ddot{y}(t)c+\dot{y}(t)d+\dot{y}(t)b=kv.b.u(t) \quad (10)$$

Set the following variables:

$$x_1(t)=y(t);\; x_2(t)=\dot{x}_1(t)=\dot{y}(t);\; x_3(t)=\dot{x}_2(t)=\ddot{y}(t);\; x_4(t)=\dot{x}_3(t)=\dddot{y}(t) \quad (11)$$

The equation (10) is rewritten as follows:

$$\dddot{y}(t)=\frac{-x_4(t).c-x_3(t).d-x_2(t).b}{a}+\frac{kv.b}{a}u(t)=g(x)+f(x).u(t) \quad (12)$$

Where:

$$g(x)=\frac{-x_4(t).c-x_3(t).d-x_2(t).b}{a}, \quad f(x)=\frac{kv.b}{a} \quad (13)$$

When there is a differential equation , to step 2 select the sliding mode variable:

$$S=\dddot{e}+\lambda_1\ddot{e}+\lambda_2\dot{e}+\lambda_3 e \quad (14)$$

$$e=y-y_d \quad (15)$$

Where: $\lambda_1,\lambda_2,\lambda_3$ is a positive constant.

Derivative of error e to derivative 3rd:

$$\dot{e}=\dot{y}-\dot{y}_d;\ddot{e}=\ddot{y}-\ddot{y}_d;\dddot{e}=\dddot{y}-\dddot{y}_d=x_4(t)-\dddot{y}_d \quad (16)$$

Take time derivative of the sliding mode variable (14), we obtain:

$$\dot{S}=\ddddot{e}+\lambda_1\dddot{e}+\lambda_2\ddot{e}+\lambda_3\dot{e} \quad (17)$$

$$\Leftrightarrow \dot{S}=\ddddot{y}(t)-\ddddot{y}_d+\lambda_1\dddot{e}+\lambda_2\ddot{e}+\lambda_3\dot{e} \quad (18)$$

Step 3: Choose the SMC laws:

The SMC consists of two terms which are equilibrium and robust terms.

$$u=u_{eq}+u_r \quad (19)$$

The equilibrium term, $u_{eq}$ is selected when we set:

$$\dot{S}=-K.S \quad (20)$$

Replace (20), (12) into (18), We have:

$$u_{eq}=\frac{1}{f(x)}(-g(x)+\ddddot{y}_d-\lambda_1\dddot{e}-\lambda_2\ddot{e}-\lambda_3\dot{e}-K.s) \quad (21)$$

$$u_r=-\frac{1}{f(x)}.\eta.sign(s) \quad (22)$$

Where $\eta\geq 0$

Replace (21), (22) into (19), we have:

$$u(t)=\frac{1}{f(x)}(-g(x)+\ddddot{y}_d-\lambda_1\dddot{e}-\lambda_2\ddot{e}-\lambda_3\dot{e}-K.s-\eta sign(S)) \quad (23)$$

Step 4: Prove stability of the controlled system

In order to prove the stability of the controlled system, a Lyapunov function is selected as follows:

$$V=S^2/2 \quad (24)$$

The time derivative of the Lyapunov function is calculated as

$$\dot{V}=S.\dot{S} \quad (25)$$

Replace (12), (18) ,(23) into (25), we have:

$$\dot{V}=-K.S^2-S.\eta.sign(S) \quad (26)$$

$$\Rightarrow \dot{V}\leq -K.S^2-\eta.S.sign(S)\leq 0 \quad (27)$$

Base on Lyapunov theory, we can state that this system in stable. Sliding variable will approach to sliding manifold in finite time. Additionally, position error and velocity error will converge to zero.

Step 5: Design a low pass filter for the reference signal:

Choose low pass filter 4rd to signal $y_d(t)$ differenible blocked to derivative 4rd . Transfer function low pass filter:

$$G_{LF}(s)=\frac{1}{(0.1s+1)^4} \quad (28)$$

**4.2. Simulation by Matlab/Simulink**

Structure of SMC system the ball in tube system by MATLAB/SIMULINK as follows:

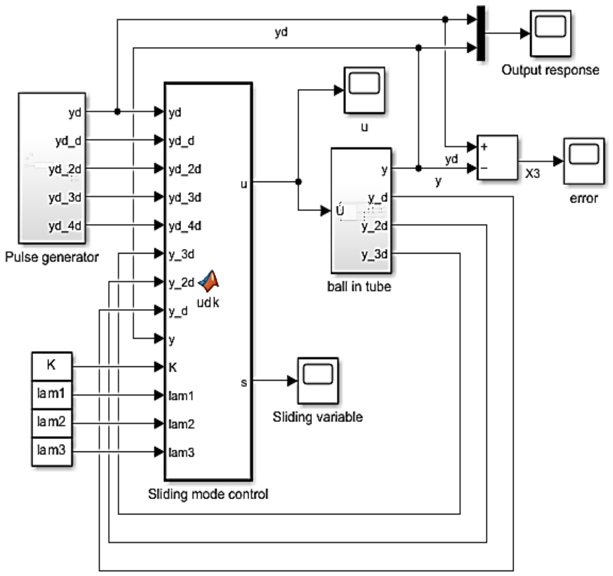


**Fig. 5.** Diagram sliding control system the ball in tube.

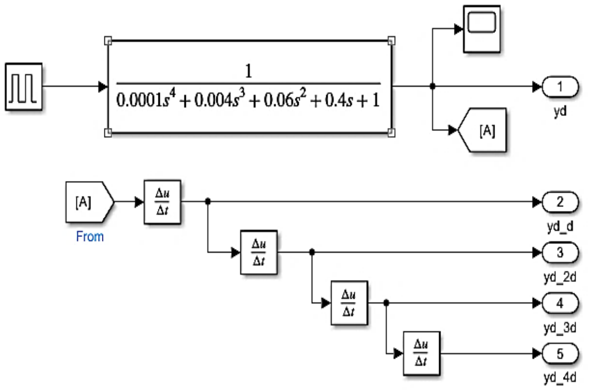


**Fig. 6.** Structure block Pulse generator.

The program in “Sliding mode control” block in Fig. 5 is designed based on equations (8), (13), (14), (15), (16), (23), and parameters in Tab. 1.

The program in MATLAB Function block (HT) in Fig. 7 is designed based on equations (8), (12), (13) and parameters in Tab. 1.

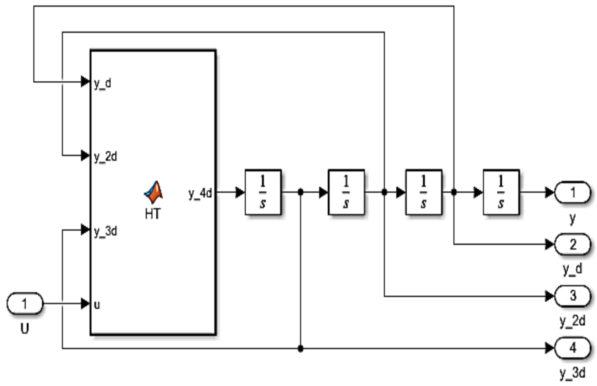


**Fig. 7.** Structure block ball in tube.

### 5. Simulation Results

About the PID controller:

To optimize parameters of PID controller, there are various methods such as GA or trial-and-error. In this paper, the authors use the trial and error method to optimize PID controller in simulation (first tuning Kp, then Ki and Kd). The authors adjust all three parameters simultaneously because they are interrelated. With PID controller parameters selected using the above method:

$K_p = 5, K_i = 0, K_d = 10$

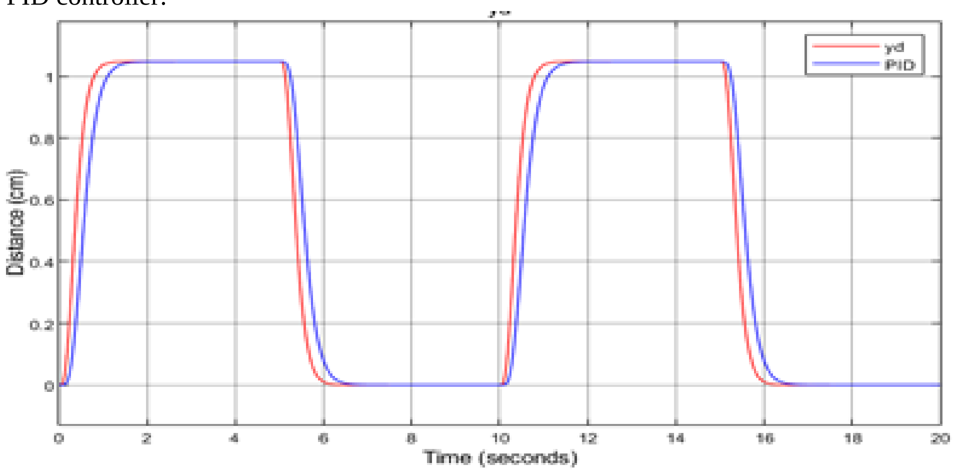


**Fig. 8.** Step trajectory tracking**.**

About SMC controller:

The “Pulse generator” signal is selected with an “Amplitude” of  , a “Period” of 10 seconds, a “Pulse width” of 50% (of the period), and a “Phase delay” of 0 seconds. Proceed to determine the parameters  of the sliding mode controller using GA. The program determines these parameters based on the following flowchart:

The simulation process is carried out with a maximum of 20 generations, “max stall generation“ set to 1000, the number of individuals used in the population is 30, the crossover rate is 0.8, and the mutation rate is 0.2. The results of some parameters obtained after being found by GA are shown in Tab. 2.

**Tab. 2.** SMC parameters

| $K$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | Generation obtained |
|---|---|---|---|---|
| 101.725 | 3.929 | 204.690 | 2.481 | 1 |
| 222.080 | 126.233 | 482.965 | 879.175 | 11 |
| 232.978 | 9.535 | 637.454 | 29.950 | 15 |

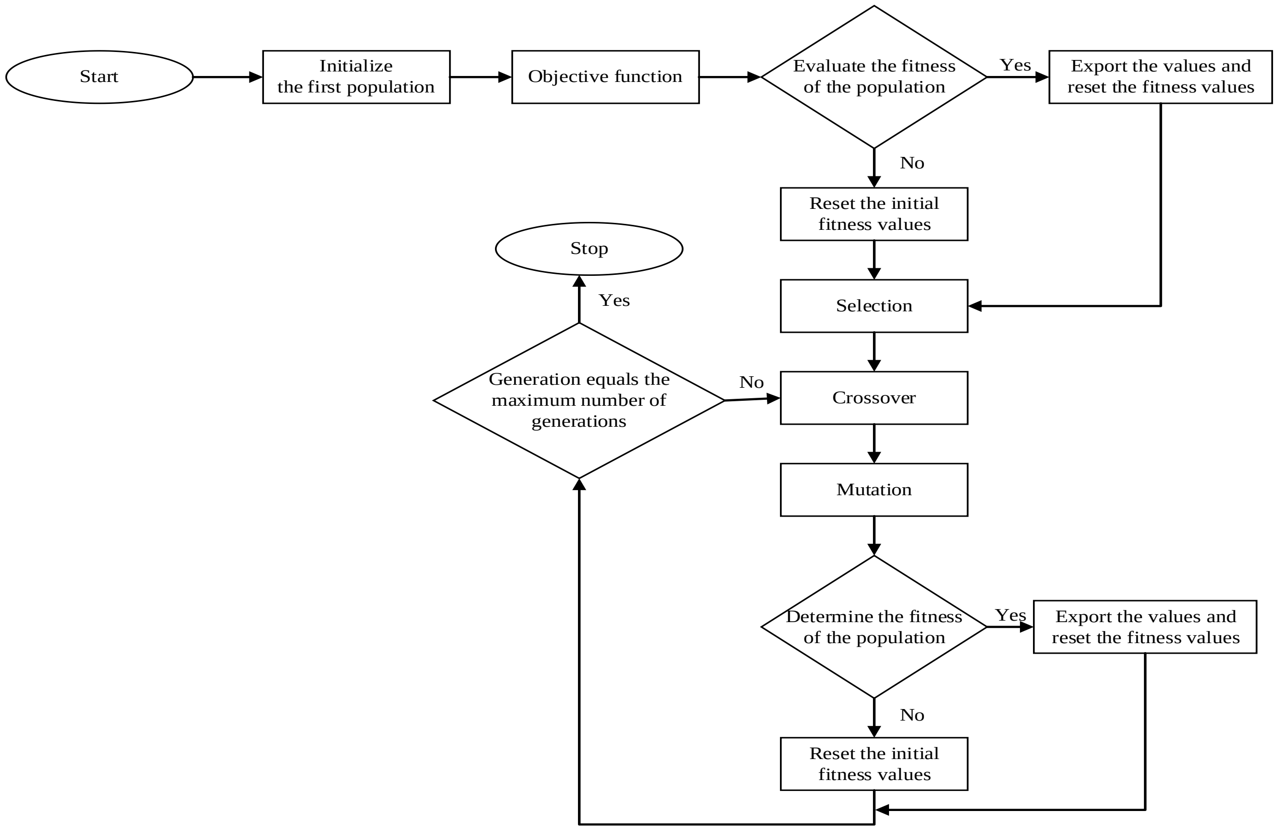


**Fig. 9.** Algorithm flowchart.

After 15 generations, found the generation with the best parameters. The output signal of the ball position in the ball in tube system corresponds to the generation achieved, displayed in the following figure:

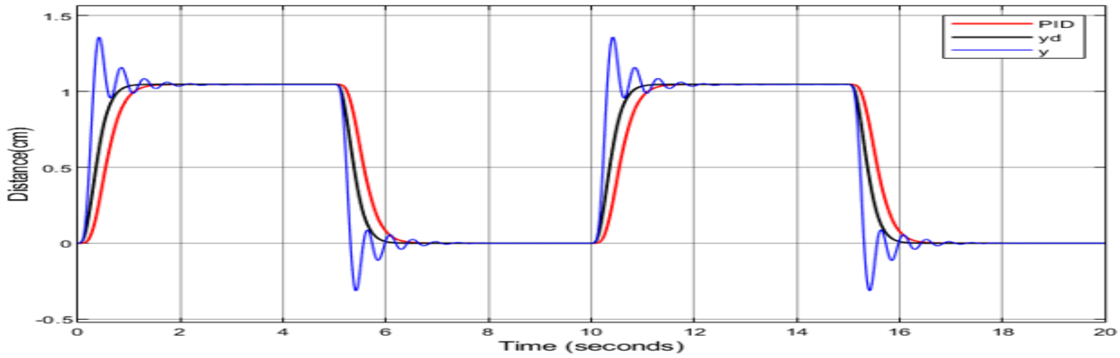


**Fig. 10.** The output response compared to the setpoint after one generation.

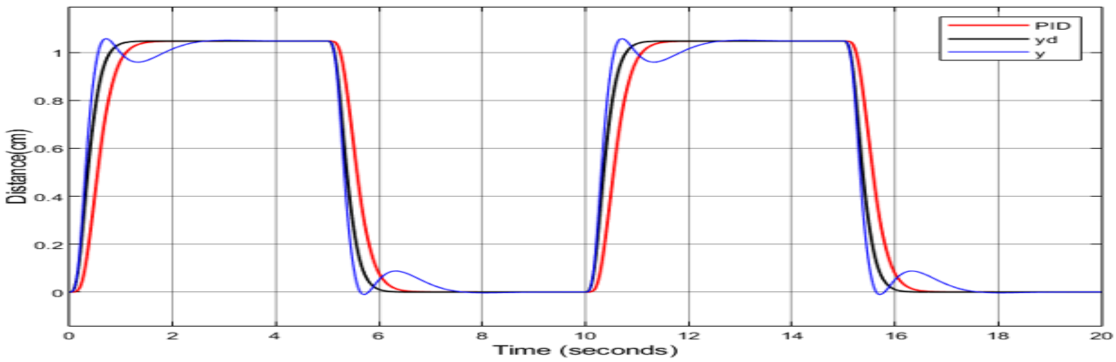


**Fig. 11.** The output response compared to the setpoint after the 11th generation.

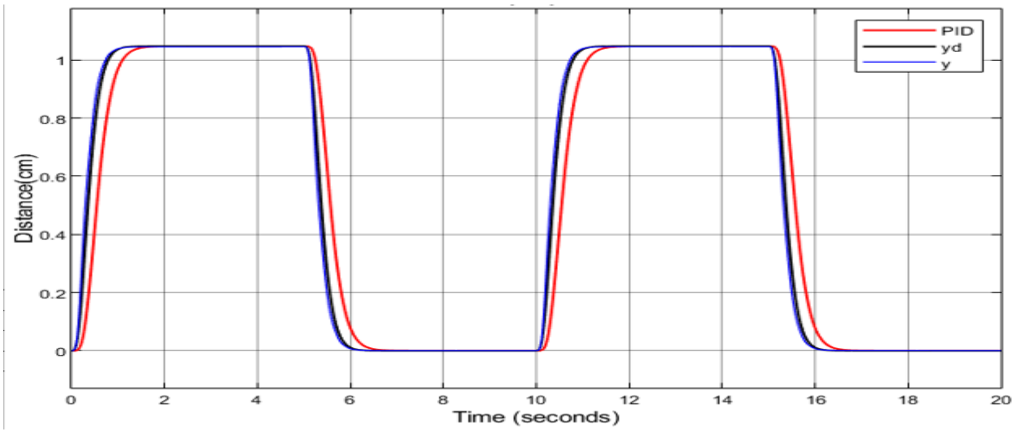


**Fig. 12.** The output response compared to the setpoint after the 15th generation.

With "PID" is the ball position after the controller PID, "y" is the ball position after the Sliding mode control, "yd" is the signal set behind the low pass filter .

After the first generation (Fig. 10), the ball position follow nearby set signal yd from the beginning.

However, the percent overshoot a high level of about 30% and Oscillate a lot around the set position. With the higher generation (Fig. 11), the ball position has been improved to follow the yd signal better. Less oscillate but still exist percent overshoot about 0.96%, significantly reduced compared to the first generation.

With the best generation (Fig. 12), the ball position stack in set signal yd. Oscillate around the set position and percent overshoot almost zero.

Through many generations of genetics (Fig. 10), (Fig. 11), (Fig. 12), the SMC controller becomes better through the generations.

We choose the best parameter set for the slide mode control corresponding to generation 15 with K=232.9780, =9.5350, =637.4540, =29.9500.

Comparing with PID control (Fig. 12), SMC control follows better signal. If PID control takes time to reach steady state, Sliding mode control allows the signal to follow from the beginning.

## 6. Conclusion

In this study, we designed a discrete PID controller and a SMC controller, and employed GA to optimize parameters of SMC controller, enabling the ball to accurately track its desired trajectory. Through simulation, the authors demonstrated the ability to progressively optimize control parameters over successive generations of GA when applied to SMC. The results also revealed the superior control performance of SMC controller compared to classical PID controller in ball-tube. Furthermore, the successful application of SMC to ball-in-tube system opens new avenues for future research in applying SMC to similar SISO system, and paves the way for experimental implementation of SMC for ball-in-tube.

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