A SURVEY OF IDENTIFICATION FOR EXPERIMENTAL DC MOTOR BY MATLAB TOOLBOX

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Abstract: This paper presents a method for estimating the parameters of a Permanent Magnet DC Motor (PMDC) using the "Parameter Estimator" tool in MATLAB. The authors also conducted simulations and experiments with the identified motor on a Rotary Inverted Pendulum (RIP) using a Linear Quadratic Regulator (LQR) controller to validate the results. The findings demonstrate that the system responds well, with a close match between simulation and experimental results.

Keywords: Parameter Estimator, PMDC Motor, RIP, LQR.

1. Introduction

DC motors are commonly used in control systems due to their linear mechanical characteristics, wide speed regulation range, and ability to handle large loads at low speeds [1]. Therefore, PMDC motors are widely applied in control models such as two-wheel balancing vehicles, inverted pendulums with inertia wheels, position control, speed control, etc. However, for most motors available on the market today, manufacturers often do not provide sufficient motor specifications, or wear and tear during usage leads to inaccurate specifications. This inaccuracy results in flawed mathematical modeling of the system, making simulation, experimentation, and controller design challenging and contributing to system instability.

In practice, there are various methods to estimate motor parameters, including Genetic Algorithm (GA) [2], Differential Evolution (DE) with two strategies, Teaching–Learning-Based Optimization (TLBO), and Artificial Bee Colony (ABC) [3]. Other methods include Ordinary Least Squares and Recursive Least Squares algorithms [4], Error Forecasting Methods, Fuzzy Modeling, Neural Network Modeling [5], etc.

In error forecasting methods, due to the mathematical characteristics of the motor equation, parameter estimation requires collecting current signals. However, in reality, the electrical speed is faster than the mechanical speed, causing the collected current signal to be incompatible with the motor speed, leading to inaccurate identification.

To ensure the estimated parameters are suitable, it's necessary to evaluate the results. Unlike the evaluation methods proposed by Marko Jesenik [3] and M. Awoda with R. Ali [6], which focus on comparing results, the authors not only validate the parameters using MATLAB but also integrate the motor model into a RIP to model the system, design an LQR controller, and conduct simulations, and experiments to evaluate identification results.

The article is structured as follows: Part 2 discusses the mathematical equations of the PMDC motor. Part 3 presents an optimization problem in MATLAB Simulink. Part 4 explains methods to collect experimental data from the motor. Part 5 outlines the steps used to estimate and validate the motor's parameters using a parameter estimator tool. Part 6 evaluates the results of RIP.

2. Model of PMDC Motor

PMDC Motor belongs to the category of independently excited DC motors. It is divided into two main parts: the field (magnet) and the armature winding. The motor structure is illustrated in **Fig. 1**

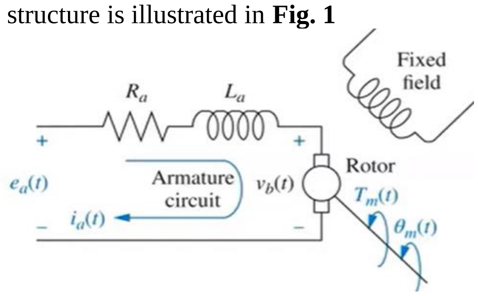

Fig. 1. Independently Excited DC Motor.

To model this, we need to divide the motor into 2 parts: the 'electrical' and the 'mechanical'. Due to Kirchhoff's law, we have the voltage balance equation in the electrical circuit component:

$$R_a i_a(t) + L_a \frac{d(i_a)}{dt} + v_b(t) = e_a(t) \quad (1)$$

where: $v_b(t) = K_b \omega(t)$ - Inductive Reactance; $\omega(t) = \frac{d\theta_m}{dt}$

Applying Newton's law to rotational motion, we have the equation of torque balance on the motor shaft:

$$J_m \frac{d\omega}{dt} + D_m \omega = T_m - T_d \quad (2)$$

where: $T_m = K_t i_a(t)$ - Motor torque

Tab. 1. Parameters of motor

Symbol	Descriptions	Unit
e_a	Armature voltage	V
L_a	Armature inductance	H
i_a	Armature current	A
v_b	Back emr	V
θ_m	Angular position of rotor	rad
T_m	Motor torque	N/m
J_m	Rotor inertia	kgm^2
D_m	Viscous friction constant	Nm/rad
K_t	Torque constant	Nm/A
K_b	Back emf constant	Vs/rad
τ	Payload torque	Nm
R_a	Armature resistance	Ω

3. Parameter Estimation as an Optimization Problem in Matlab/Simulink

When performing parameter estimation of a PMDC motor, software formulates an optimization problem [7]. The solution to this problem is set of estimated parameter values of the PMDC motor. This problem includes:

- x - Design variables: The parameters of the PMDC motor model and the initial state to be estimated.
- $x_{\min} \le x \le x_{\max}$ Bounds: Limits on the estimated parameter values. For PMDC motor parameter estimation must be $0 \le x \le x_{\max}$.
- $F(x)$ - Objective function: A function calculating the measure of difference between simulated $(\omega_{sim}(t))$ and experimental $(\omega_{ref}(t))$ motor speed responses. It is also known as the cost function or estimation error. The cost function is Sum squared error:

$$F(x) = \sum_{t=0}^{n_t} e(t)^2 \quad (3)$$

where n is the number of samples

$$e(t) = \omega_{ref}(t) - \omega_{sim}(t) \quad (4)$$

Optimization solver tunes the values of designed variables to satisfy specified objectives and constraints. Optimization solver applied to the PMDC Motor here is Nonlinear Least Squares (NLS). This choice is made because the cost function is not scalar and number of error residuals does not change from one iteration to another.

Objective of NLS method is finding a vector x inimizing cost function $F(x)$:

$$\min_x \|F(x)\|_2^2 = \min\left(f_1(x)^2 + f_2(x)^2 + ... + f_n(x)^2\right) \quad (5)$$

where: $f_1(x), f_2(x), ..., f_n(x)$ scalar-valued nonlinear function of residual of data point $i = 1, 2, ..., n$ in which n is the number of samples, x is vector of variables representing the motor parameters to be estimated. Various algorithms are used in the MATLAB Toolbox to solve equations(5), including:

- Levenberg-Marquardt algorithm [8]. This algorithm combines the Gauss-Newton method with Gradient Descent to search for the optimal point x.
- Trust-Region-Reflective algorithm[9]. This method restricts the search range within a confidence region x to ensure algorithm stability.

In this parameter identification problem for the PMDC motor, the authors use the Levenberg-Marquardt method to solve the numerical equations (5)

Optimization stopping criterion is decided by several factors: if two successive parameters change by less than the selected parameter tolerance $(|x_{i+1} - x_i| < \varepsilon_1)$, or if cost function values change by less than selected function tolerance $(|F_{i+1} - F|_1 < \varepsilon_2)$, or if maximum number of iterations is reached. By adjusting these parameters, optimization can continue searching for a more accurate solution [6].

4. Input-Output Motor Data Estimation

The process of collecting supply voltage and angular velocity data through communication between a computer and a microcontroller is demonstrated in **Fig. 2**

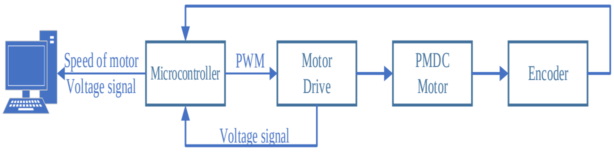

Fig. 2. Block diagram of experimental voltage and speed data.

We collect two datasets. The first dataset is used to estimate parameters of PMDC motor with voltage signals represented as a Fourier series:

$$V = 3(\sin(0.4\pi t) + \sin(0.6\pi t) + \sin(2\pi t) + \sin(1.2\pi t)) \ (V) \quad (6)$$

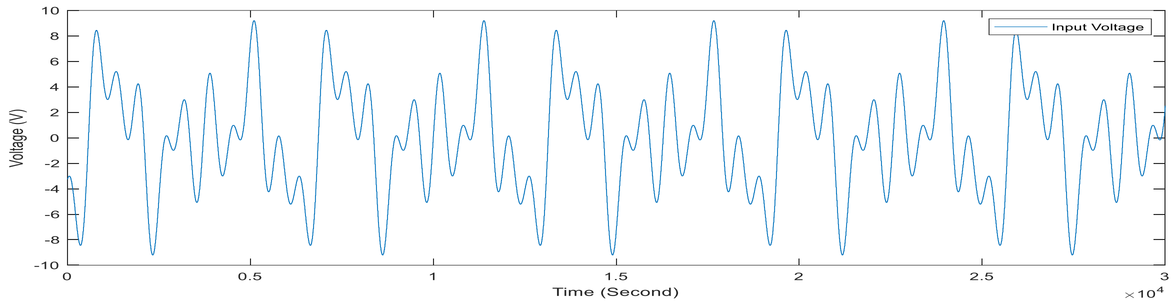

Fig. 3. Fourier input signal.

Second dataset is used to evaluate estimation results with voltage signals represented as a sinusoidal waveform:

$$V = 12\sin(2\pi 0.15t)\ (V) \tag{7}$$

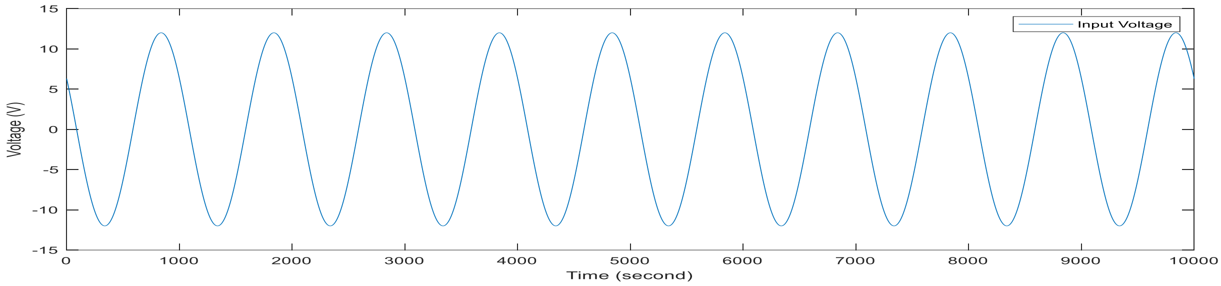

Fig. 4. Sinusoidal input signal.

With such input voltage signals to the motor, the authors use an encoder mounted on the motor shaft to read the rotor angle with a sampling period $T_s = 0.01(s)$. The signal is sent back to the microcontroller to calculate the rotor angular velocity and transmit it to the computer.

The rotor angular velocity when the Fourier input signal is represented is shown in **Fig. 5**.

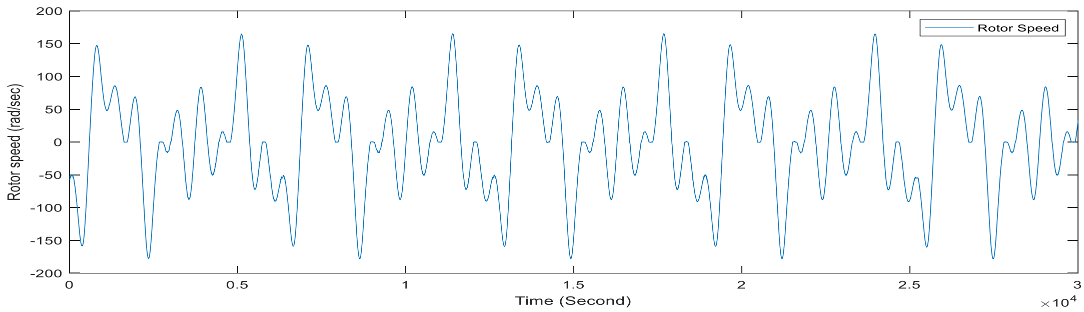

Fig. 5. Rotor speed with fourier input signal.

The rotor angular velocity when the sinusoidal input signal is represented is shown in **Fig. 6**.

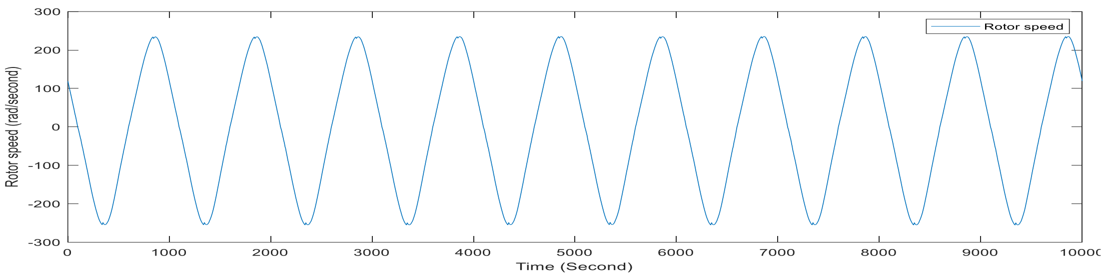

Fig. 6. Rotor speed with Sinusoidal input signal.

Collected data is stored as a structure with time in MATLAB to store the experimental data for estimation and validation.

5. The Identification Process

After collecting data, we proceed to simulate motor system in MATLAB Simulink. The motor is described by mathematical equations (1) and (2) with initial parameters of PMDC motor. These parameters are obtained from another motor with known specifications and loaded into workspace.

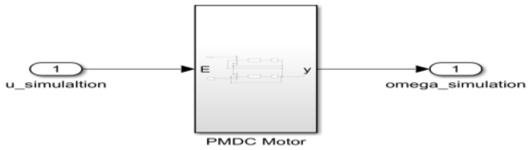

Fig. 7. Simulating the motor system in Matlab Simulink.

5.1. Estimate Parameter PMDC Motor.

To estimate the parameters of the PMDC motor, we use Parameter Estimator toolbox for Matlab. Here, we use Matlab 2023a. All steps follow the guidelines provided by MathWorks as (from [10]):

a. In the Simulink, we select **Apps** > **Parameter estimator.**

b. Selecting Parameters for Estimation. Estimation parameters are chosen by clicking **Select Parameters** in **Parameter Estimator** tab. For PMDC Motor, we obtain six parameters of motor model: $D_m, J_m, K_t, K_b, R_a, L_a$. By knowing from our physical insight that these parameters cannot be negative, we set their lower limits to zero.

c. Load estimation data by adding a new experiment. Select inputs as the voltage signal with a Fourier series and the corresponding output as the rotor speed.

d. Add progress plots by clicking **Add Plot** on the Parameter Estimation tab, in Experiment Plots select **Exp**. In Cost Function select **Sum Squared Error**. Select **More Option** > **Optimization**, in tag Estimation Options select method **Nonlinear least squares**, **Trust-Region-Reflective** algorithm, click ok, and run the estimation.

Fig. 8 shows experimental data overlaid with simulated data. Simulated data comes from the model with six estimated parameters. Results of estimation show that output matches.

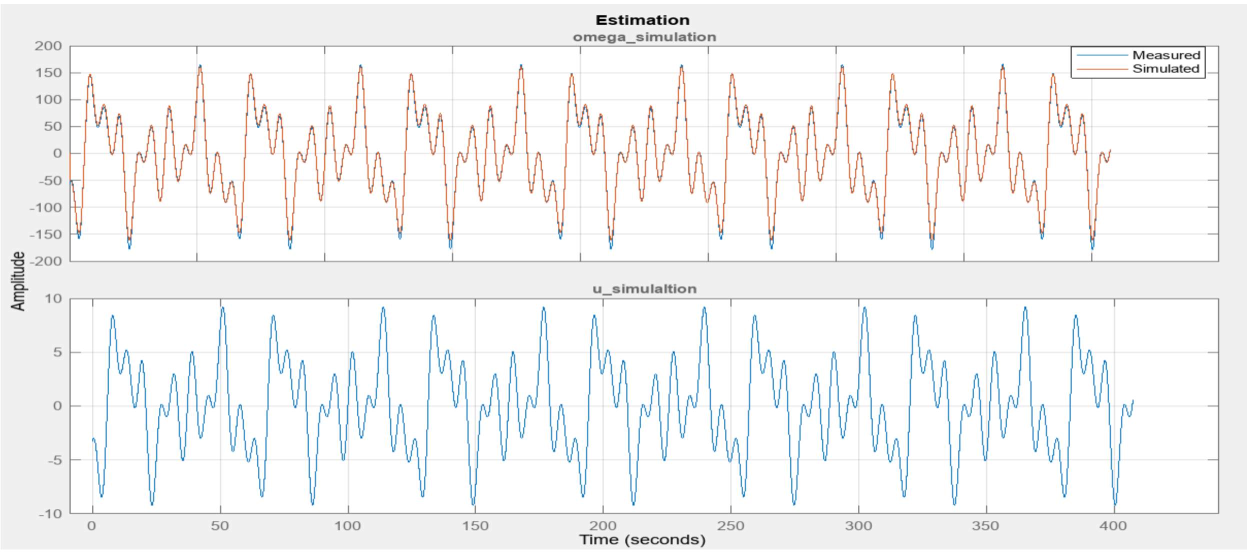

Fig. 8. Relationship between Simulated and experimental outputs with the Fourier input signal.

The estimated parameters are shown as

$$L_a = 0.007703\,(H);\; J_m = 0.0069575\ (\text{Kgm}^2);$$
$$D_m = 0.037751\ (\text{Nm/rad});\; K_t = 0.54897\ (\text{Nm/A}); \quad (8)$$
$$K_b = 0.024502\ (\text{Vs/rad});\; R_a = 0.47345\,(\Omega)$$

5.2. Validation Parameter PMDC Motor.

Successful estimation will not only match experimental data that was used for estimation but also other data sets experimentally collected [10]. Therefore, we added an evaluation step with a sinusoidal input signal. The validation in the Parameter Estimator follows the following steps:

a. Load the validation data by selecting **Validation** > **New Experiment**. Select the inputs as the sinusoidal voltage signal and the corresponding output as the rotor speed.

b. Right-click on the added dataset, select **Use Experiment for Validation**, then choose **Validate**.

The results after validation are presented on **Fig. 9**. Validation demonstrates that the model effectively handles sinusoidal input signal from the validation data, indicating that model parameters were successfully estimated.

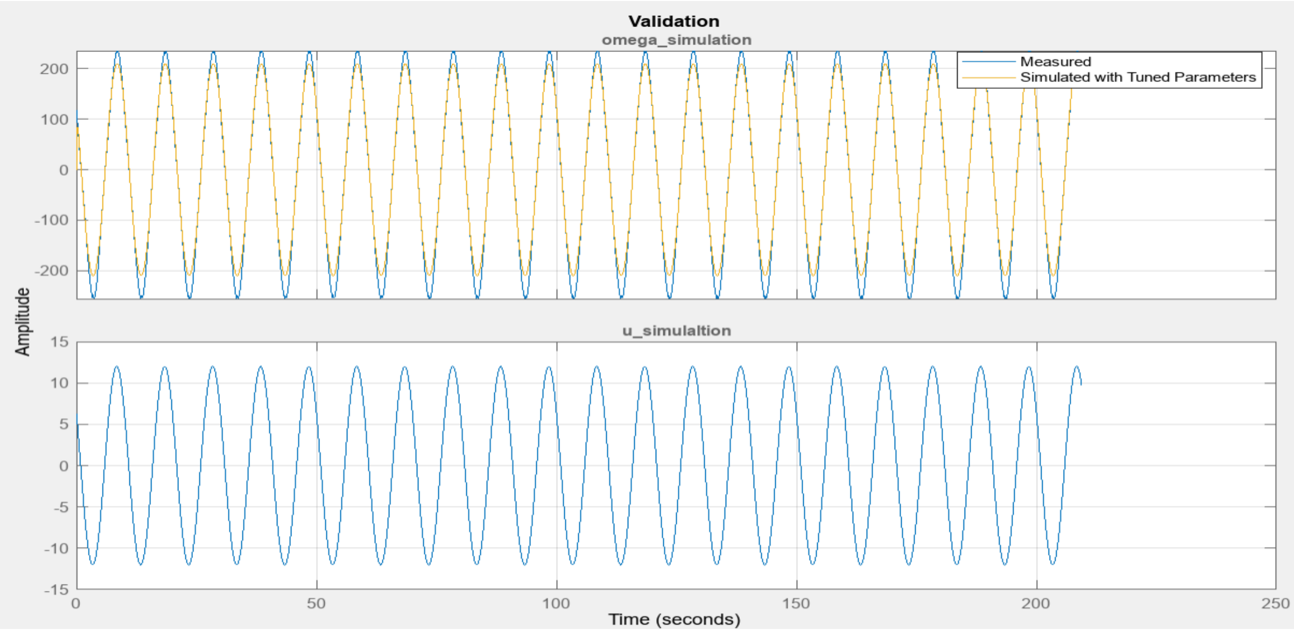

Fig. 9. The relationship between the simulated and experimental outputs with the Sinusoidal input signal.

6. Evaluate the PMDC Motor Parameter using LQR Controller on RIP

Because the objective of parameter estimation for the motor is to model and design a controller for the system. Therefore, to evaluate the estimated parameters for the motor, the authors will design an LQR controller on the represented RIP in **Fig. 10**.

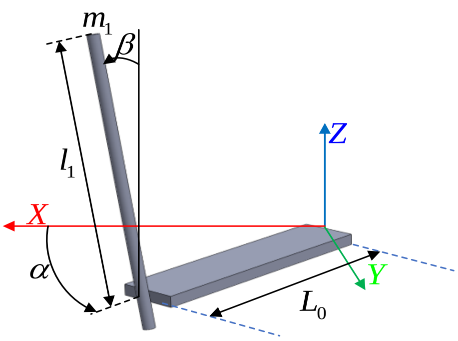

Fig. 10. RIP model.

According to [5], we have mathematical equations describing RIP system as follows:

$$\begin{bmatrix} J_0 + m_1L_0^2 + m_1l_1^2\sin^2\beta & -m_1L_0l_1\cos\beta \\ -m_1L_0l_1\cos\beta & J_1+m_1l_1^2\end{bmatrix}\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\end{bmatrix} + \begin{bmatrix} C_0+\frac{1}{2} m_1l_1^2\dot{\beta}\sin 2\beta & m_1L_0l_1\dot{\beta}\sin\beta+\frac{1}{2} m_1l_1^2\dot{\alpha}\sin 2\beta \\ -\frac{1}{2} m_1l_1^2\dot{\alpha}\sin 2\beta & C_1\end{bmatrix}\begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\end{bmatrix} + \begin{bmatrix}0\\ -m_1gl_1\sin\beta\end{bmatrix} = \begin{bmatrix}\tau\\ 0\end{bmatrix} \qquad (9)$$

The control signal here is torque of motor (τ). It needs to be converted into voltage to fit the real system. Torque produced by DC motor is defined by:

$$\tau = \frac{K_tV}{R_m} + \left(\frac{-K_tK_b}{R_m} - D_m\right)\dot{\alpha} - J_m\ddot{\alpha} \qquad (10)$$

Tab. 2. The parameters and variables of the system.

Values	Description	Value	Unit
m_1	Mass of pendulum	0.05322	Kg
l_1	Length of pendulum stick	0.33	m
L_0	Length of pendulum arm	0.22	m
J_0	Inertia moment of arm	0.00817	kgm²
J_1	Inertia moment of pendulum	0.00912	kgm²
g	Gravitational acceleration constant	9.81	m/s²
α	Pendulum arm angle		rad
β	Pendulum angle		rad
V	Armature voltage		V
τ	Torque applied to the arm axis		Nm
K_t	Torque constant	0.54897	Nm/A
K_b	Back emf constant	0.024502	Vs/rad
R_a	Armature resistance	0.47345	Ω
J_m	Moment of inertia of rotor	0.0069575	kgm²
D_m	Viscous friction constant	0.037751	Nms/rad

From (9), (10). we obtain the linear state equations of RIP at the operating point:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \qquad (11)$$

where: $x = \begin{bmatrix}\alpha & \dot{\alpha} & \beta & \dot{\beta}\end{bmatrix}^T$; $u = V$; $y = \begin{bmatrix}\alpha & \beta\end{bmatrix}^T$

With system parameters from Tab. 2, the matrices A, B, C at the $x = 0$ operating point are as follows:

$$A=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & -4.537 & 4.78 & -0.004\\ 0 & 0 & 0 & 1\\ 0 & -2.013 & 21.908 & -0.02\end{bmatrix}; \quad (12)$$

$$B=\begin{bmatrix}0 & 72.49 & 0 & 32.17\end{bmatrix}^T;\ C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

Because we use a microcontroller to control the system with a sampling time T_s=10ms, discrete-time matrices A, B and the weighting matrices for the LQR controller have the following values:

$$A_d=\begin{bmatrix}1 & 0.0098 & 0.0002 & 0\\ 0 & 0.9556 & 0.0467 & 0.0002\\ 0 & -0.0001 & 1.0011 & 0.01\\ 0 & -0.0197 & 0.2187 & 1.0009\end{bmatrix}; \quad (13)$$

$$B_d=\begin{bmatrix}0.0036 & 0.7087 & 0.0016 & 0.3146\end{bmatrix}^T;$$

$$Q=\begin{bmatrix}50 & 0 & 0 & 0\\ 0 & 0.01 & 0 & 0\\ 0 & 0 & 10000 & 0\\ 0 & 0 & 0 & 0.001\end{bmatrix};\ R=0.7$$

From (13), matrix K is obtained as

$$K=\begin{bmatrix}-5.47 & -4.93 & 97.51 & 13.3\end{bmatrix} \quad (14)$$

With the mathematical equations (9), (10) and the system parameters from **Tab. 2**, the authors simulated RIP on Matlab Simulink **Fig. 11**.

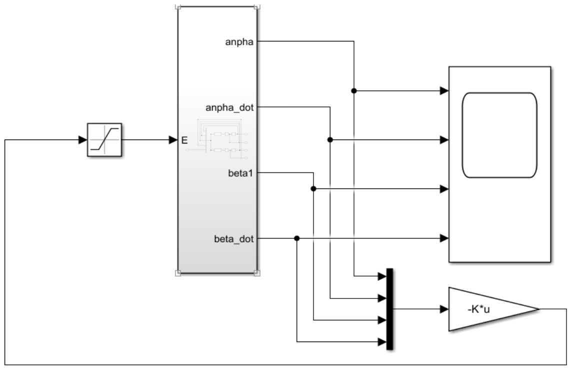

Fig. 11. Simulation of the RIP system.

$$x=\begin{bmatrix}\alpha & \dot{\alpha} & \beta & \dot{\beta}\end{bmatrix}=\begin{bmatrix}0.18 & 0 & -0.02 & 0\end{bmatrix}$$

System response is shown in **Fig. 12**

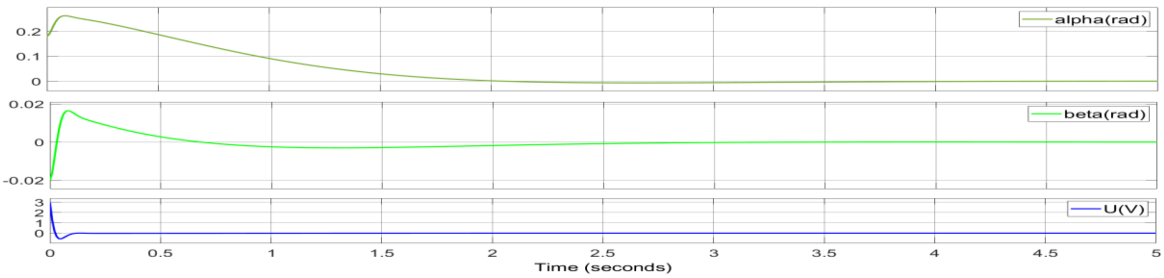

Fig. 12. The output response of the system in the simulation.

Output experimental response is shown in Fig. 13 and system response is shown in Tab.3

Tab. 3. System response

	Pendulum angle (rad)	Arm angle (rad)	Control voltage (V)
Error	[0.017;0.17]	[-0.01; 0.003]	[-0.623;1.3]
Average error	0.05	0.003	0.2

With initial state angles during simulation:

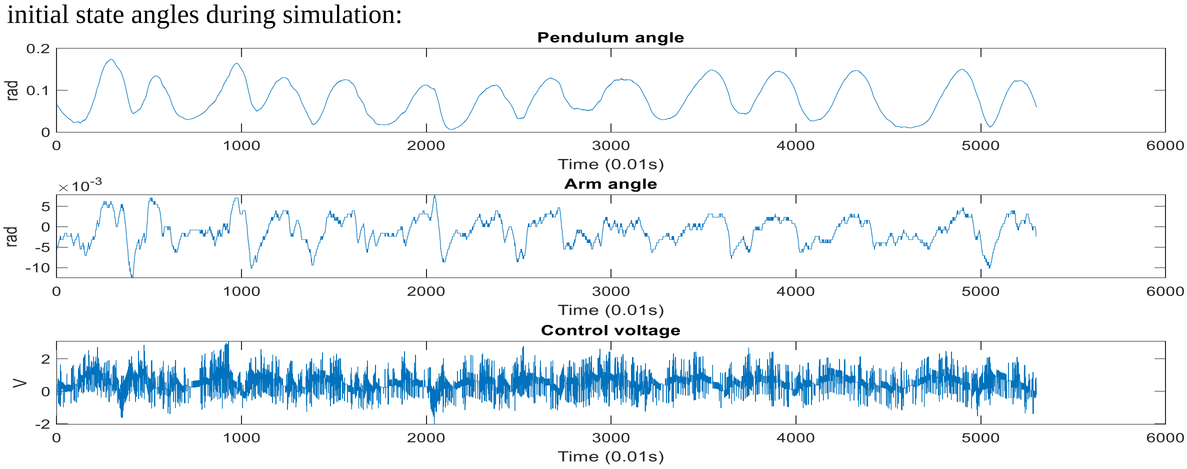

Fig. 13. The output experimental response of the system.

7. Conclusion

To solve parameter estimation problem for motor as an optimization task in Matlab Simulink, we utilized Nonlinear Least Squares method Trust-Region-Reflective algorithm. We built a Simulink model with dynamic equations of the PMDC motor and used parameters from another motor to simulate the initial motor system. Estimation was performed using a Fourier input signal, followed by validation with a sinusoidal input signal. Simulation results of the motor system with estimated parameters closely matched experimental results, demonstrating suitability of PMDC motor parameters. Since parameter estimation aims to design a controller for improved system stability, we applied estimated motor to a RIP by using LQR controller for further validation. The consistency between simulation and experimental results confirms that estimated parameters are appropriate for the system.

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