APPLICATION OF FUZZY-ANFIS CONTROLLER FOR BALL ON WHEEL SYSTEM

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Abstract: The ball-on-wheel (BoW) is a nonlinear single input-multi output (SIMO) system consisting of a ball located on a wheel. The system's challenge is to balance ball at the highest point on the wheel. In this paper, we propose a method of applying ANFIS toolbox in imitating a successful controller (in this case, it is PD controller). This process creates a similar fuzzy controller which controls well this system. Therefore, through this research, ANFIS toolbox shows the ability in imitating an expert‘s control law through collecting data of operations.
Keywords: ball-on-wheel, nonlinear, SIMO system, Fuzzy controller, ANFIS toolbox.

1. Introduction

BoW is a SIMO model used in laboratory to study control algorithms. This system has been successfully modeled and controlled using the state feedback method [1]. Additionally, PID controller has been validated for stabilizing the ball on wheel system [2]. Thence, controllers act well when being applied to this model. However, if we act as a person who cannot control system, but, collect adequate data from a successful operation process of experts. A method is suggested that with that data, we can re-build a similar controller which has at least quality as the former controller. In this paper, we propose using ANFIS fuzzy which is created from ANFIS toolbox of Matlab/Simulink [3]. ANFIS is a combination of Neural Networks and Fuzzy Logic, providing accuracy for nonlinear systems [4].

In this study, we use it to imitate a successful PID controller when operating on BoW.

2. Modeling BoW

Mathematical model of BoW is shown in Fig. 1 *[1]*.

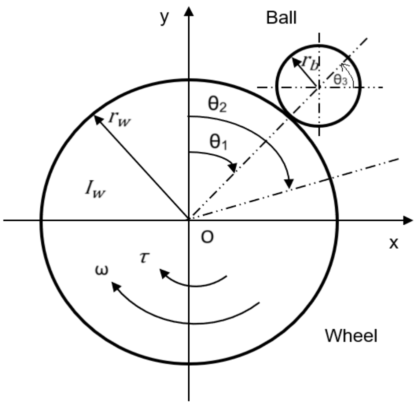

Fig. 1. Mathematical model of BoW.

Euler - Lagrange equation form of the model is:

$$Q = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} \quad (1)$$

where, Q is general force function, q is general coordinate, L is the Lagrangian operator:

$$T - V = L \quad (2)$$

For this system, V is the potential energy T is the kinetic energy, then q is defined as:

$$q = \begin{bmatrix}\theta_1 & \theta_2\end{bmatrix}^T \quad (3)$$

Angular displacement of wheel is equivalent to angle of rotation between y-axis and the line that connects the center of the ball and the wheel. Thus, value of Q can be determined by:

$$Q = \begin{bmatrix}0 & \tau\end{bmatrix}^T \quad (4)$$

where, τ is torque acting on wheel, momentum of ball is calculated as:

$$T_b = \frac{m_b(r_\omega + r_b)^2\theta_1^2}{2} + \frac{I_b\theta_3^2}{2} \quad (5)$$

in which m_b is the mass of the ball, r_w is the radius of the wheel, r_b is the radius of the ball, and θ_3 is the angular displacement of the ball's center relative to the vertical axis. Then the moment of inertia of the ball is calculated using the formula:

$$I_b = 2m_b r_b^2/5 \quad (6)$$

Kinetic energy of wheel is:

$$T_\omega = I_\omega \dot{\theta}_2^2/2 \quad (7)$$

Next, total kinetic energy of system is:

$$T = T_\omega + T_b$$
$$= \frac{1}{2} I_\omega \dot{\theta}_2^2 + \frac{1}{2} m_b (r_\omega + r_b)^2 \dot{\theta}_1^2 + \frac{1}{2}(\frac{2}{5} m_b r_b^2)\dot{\theta}_3^2 \quad (8)$$

When we rotate the disc and place the ball on the disc, a constraint condition is created between the disc and the ball. In our reference system, we use 'V_{C_w/O_w}' to represent velocity of contact point 'Cw' with respect to center of wheel 'O_w', we can use following notation:

$$V_{C_\omega/O_\omega} = \dot{\theta}_2 r_\omega e \quad (9)$$

where e is a unit vector tangent to point of contact and increases in the positive direction of θ_2. And V_{C_b/O_b} represents contact point velocity C_b with respect to the center of the ball O_b as observed from the stationary reference system. Expressed by the formula:

$$V_{C_b/O_b} = \dot{\theta}_3 r_b e \quad (10)$$

Assuming there is no slipping during the rolling motion of the ball, the contact point C_b is momentarily at rest relative to the contact point C_w at that instant. We have velocity of two contact points above as:

$$V_{C_b/C_\omega} = 0 \quad (11)$$

Based on the mathematical model in the figure, we can derive the relative speed of the ball's center to the wheel's center as:

$$V_{O_b/O_w} = \dot{\theta}_1 (r_\omega + r_b) e \quad (12)$$

From *(9)* and *(11)*, we get:

$$V_{O_b/O_\omega} = V_{O_b/C_b} + V_{C_b/C_\omega} + V_{C_\omega/O_\omega} = -\dot{\theta}_3 r_b e + \dot{\theta}_2 r_\omega e \quad (13)$$

We again have the rolling condition from two equations *(12)* and *(13)*:

$$r_\omega \dot{\theta}_2 - (r_\omega + r_b)\dot{\theta}_1 = r_b \dot{\theta}_3 \quad (14)$$

One thing to note is that we cannot measure $\dot{\theta}_3$ directly. To control the feedback, we can reduce $\dot{\theta}_3$ from *(14)* to the accuracy of $\dot{\theta}_1$ and $\dot{\theta}_3$, we use *(14)* again to have:

$$T = \frac{I_\omega \dot{\theta}_2^2}{2} + \frac{m_b (r_\omega + r_b)^2 \dot{\theta}_1^2}{2} + \frac{m_b (r_\omega \dot{\theta}_2 - r_\omega \dot{\theta}_1 - r_b \dot{\theta}_1)^2}{5} \quad (15)$$

Total potential energy is:

$$V = m_b g \cos\theta_1 (r_\omega + r_b) \quad (16)$$

where g is gravitational accelaration = 9.81 m/s^2.

Larangian function is:

$$L = T - V = \frac{I_\omega \dot{\theta}_2^2}{2} + \frac{m_b (r_\omega + r_b)^2 \dot{\theta}_1^2}{2} + \frac{m_b (r_\omega \dot{\theta}_2 - r_\omega \dot{\theta}_1 - r_b \dot{\theta}_1)^2}{5} - m_b g (r_\omega + r_b)\cos\theta_1 \quad (17)$$

Then, we obtain :

$$\frac{\partial L}{\partial \theta_1} = m_b \delta (r_\omega + r_b)\sin\theta_1 \quad (18)$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = (\frac{7}{5} r_\omega^2 m_b + \frac{14}{5} r_\omega r_b m_b + \frac{7}{5} r_b^2 m_b)\dot{\theta}_1 + -(\frac{2}{5} r_\omega^2 m_b + \frac{2}{5} r_\omega r_b m_b)\dot{\theta}_2 \quad (19)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = \left(\frac{7}{5} r_\omega^2 m_b + \frac{14}{5} r_\omega r_b m_b + \frac{7}{5} r_b^2 m_b\right)\ddot{\theta}_1 + + \left(-\frac{2}{5} r_\omega^2 m_b - \frac{2}{5} r_\omega r_b m_b\right)\ddot{\theta}_2 \quad (20)$$

$$\frac{\partial L}{\partial \theta_2(t)} = 0 \quad (21)$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = \left(-\frac{2}{5} r_\omega^2 m_b - \frac{2}{5} r_\omega r_b m_b\right)\dot{\theta}_1 + \left(I_\omega + \frac{2}{5} r_\omega^2 m_b\right)\dot{\theta}_2 \quad (22)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = \left(-\frac{2}{5} r_\omega^2 m_b - \frac{2}{5} r_\omega r_b m_b\right)\ddot{\theta}_1 + + \left(I_\omega + \frac{2}{5} r_\omega^2 m_b\right)\ddot{\theta}_2 \quad (23)$$

$$\left(7r_b + 7r_\omega\right)\ddot{\theta}_1 - 2r_\omega \ddot{\theta}_2 - 5g\sin\theta_1 = 0 \quad (24)$$

$$\left(-\frac{2}{5} r_\omega^2 m_b - \frac{2}{5} r_w r_b m_b\right)\ddot{\theta}_1 + \left(I_\omega + \frac{2}{5} r_\omega^2 m_b\right)\ddot{\theta}_2 = \tau \quad (25)$$

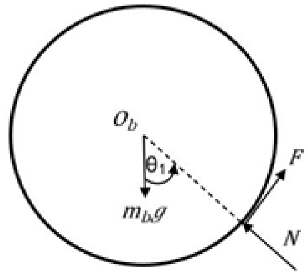

Fig. 2. Free-body diagram of the ball.

Equations (24) and (25) are only valid when centripetal force is sufficient to maintain the circular motion of the ball on wheel. Vice versa, ball may fall off wheel. Considering the diagram in Fig. 2, according to Newton's third law, the centripetal force can be mathematically described by the following equation:

$$m_b g \cos\theta_1 = m_b (r_\omega + r_b)\dot{\theta}_1^2 + N \quad (26)$$

where, N is the normal force. Force provided by $m_b \delta \cos\theta_1 - N$ is necessary to sustain the ball's motion on wheel. Ball will fall off the wheel when N = 0. Hence, in order to keep the ball on the disc, it is necessary that:

$$N = m_b\delta\cos\theta_1 - m_b(r_\omega + r_b)\dot{\theta}_1^2 > 0 \tag{27}$$

Due to the negligible viscous friction coefficient, the motor model can be expressed by the following equation:

$$\tau = \frac{K_m u}{R_a} - \frac{K_m^{\,2}\dot{\theta}_2}{R_a} \tag{28}$$

where, τ is torque control, u denotes the voltage control, K_m is the motor constant, and R_a represents the motor armature resistance. State vector of variables is defined as:

$$x = \left[x_1 x_2 x_3 x_4\right]^T = \left[\theta_1\dot{\theta}_1\theta_2\dot{\theta}_2\right]^T. \tag{29}$$

Equations (24), (25) and (28) allow us to define the state space of both the system and the disc in the base frame. By utilizing equation (29), we can represent this state space in an alternative form:

$$x = f(x) + g(x)u, \tag{30}$$

in which:

$$f(x) = \begin{bmatrix} x_2 \\ ax_4 + b\sin x_1 \\ x_4 \\ px_4 + q\sin x_1 \end{bmatrix}; g(x) = \begin{bmatrix} 0 \\ c \\ 0 \\ r \end{bmatrix} \tag{31}$$

and a, b, c, q, p, r are defined as:

$$a = -\frac{2r_\omega K_m^2}{R_a(7I_\omega + 2r_\omega^2 m_b)(r_b + r_\omega)};$$

$$b = \frac{\delta(5I_\omega + 2r_\omega^2 m_b)}{(7I_\omega + 2r_\omega^2 m_b)(r_b + r_\omega)};$$

$$c = \frac{2r_\omega K_m}{R_a(7I_\omega + 2r_\omega^2 m_b)(r_b + r_\omega)}; \quad p = -\frac{7K_m^2}{R_a(7I_\omega + 2r_\omega^2 m_b)};$$

$$q = \frac{2\delta r_\omega m_b}{7I_\omega + 2r_\omega^2 m_b}; \quad r = \frac{7K_m}{R_a(7I_\omega + 2r_\omega^2 m_b)}; \quad ar = cp.$$

System parameters are chosen as in Tab. ***1***. These parameters are measured from real model in [5]

Tab. 1. Simulation parameters for the system.

Parameter	Value
Wheel's moment of inertia	$1.71\text{x}10^{-3}$ (kgm^2)
Wheel radius	0.075 (m)
Ball mass	0.059 (kg)
Ball radius	0.03125 (m)
Motor armature resistance	0.656 (Ω)
Motor constant	0.66 (Nm/A)

3. Control Algorithm

3.1 PID Algorithm

PID controller of BoW includes two small PID controllers: PID1 controlled ball angle and PID2 controlled wheel angle. The scheme of PID controller is shown in Fig. 3.

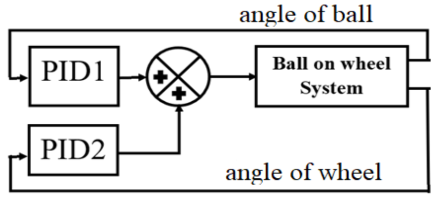

Fig. 3. The PID control scheme of BoW [5]

PID1 and PID2 controller parameters are: Kp1, Ki1, Kd1 and Kp2, Ki2, Kd2.

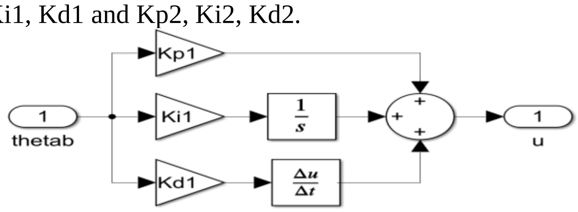

Fig. 4. The scheme PID1 controller.

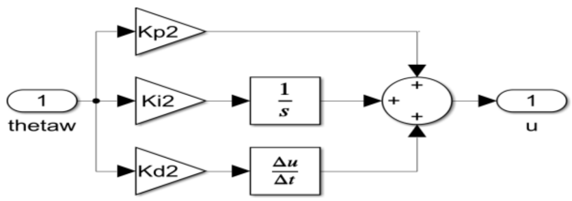

Fig. 5. The scheme PID2 controller.

3.2. Adaptive Neuro-Fuzzy Inference System (ANFIS)

A fuzzy controller is built from the PID controller just built above to control BoW.

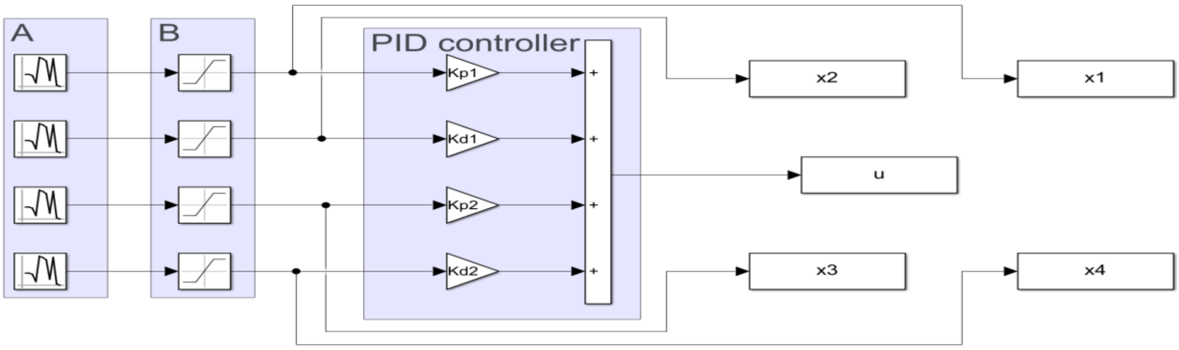

Fig. 6. Experimental data acquisitioning scheme.

Step 1: Collect data from PID controller.

Explanation of the blocks in Fig. 6:

A: Random number blocks.

B: Saturation blocks.

Step 2: Assign the collected data to the matrix and save to the workspace using the command:

“$data = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad u]$”

Step 3: Use ANFIS toolbox from Matlab.

Launch ANFIS with the command: “anfisedit”.

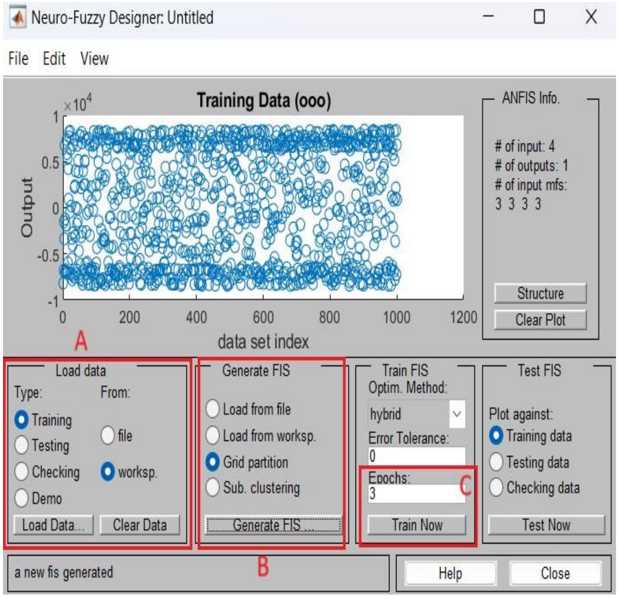

Fig. 7. ANFIS toolbox interface.

Explanation of the blocks in Fig. 7:

A: Block load data: click “load data” button and import variable name “data”(saved in step 2).

B: Block Generate FIS: click “Generate FIS” button to assign type and number of membership function(Fig. 8).

C: Block train: choose training epochs.

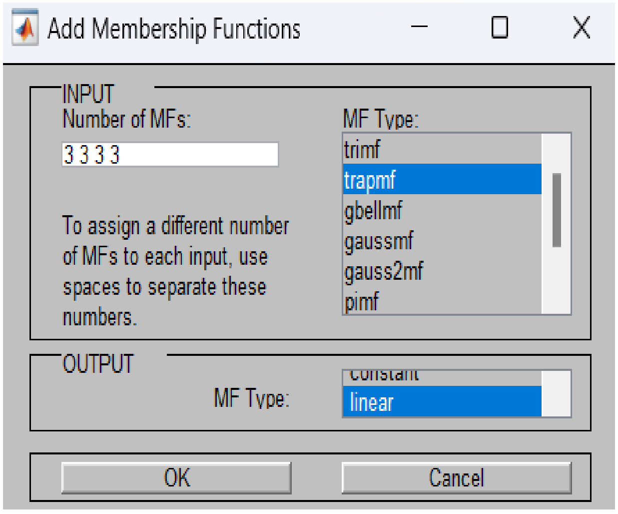

Fig. 8. Input and output membership functions.

Step 4: Export file “fis” to create Fuzzy controller.

Click “file”, “export”, “to file”, select path to save file.

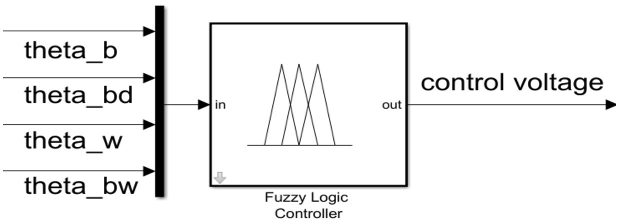

Fig. 9. The fuzzy control scheme of BoW

4. Simulation Results

The initial values of Ball-on-wheel system are:

$$x_{1\_init} = 0.01(rad); \qquad x_{2\_init} = 0.02(rad/s); \tag{32}$$

$$x_{3\_init} = 0.03(rad); \; x_{4\_init} = 0.04(rad/s)$$

PID controller parameters are selected as follow:

$$K_{p1} = -9380; \quad K_{i1} = 0; \quad K_{d1} = -970; \tag{33}$$

$$K_{p2} = 66; \; K_{i2} = 0; \; K_{d2} = 88$$

The Fuzzy controller is created by choose type, number of membership functions:

Input

Number of membership functions : “3 3 3 3”.

Membership functions type: “trapmf”.

Output

Membership functions type: “linear”.

Epochos: “3”.

Simulation results are shown in ***Fig. 10*** and ***Fig. 11*** below

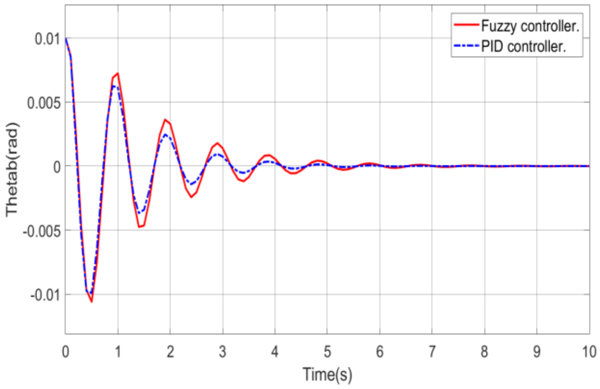

Fig. 10. Comparison of ball angle proposed for Fuzzy controller and PID controller.

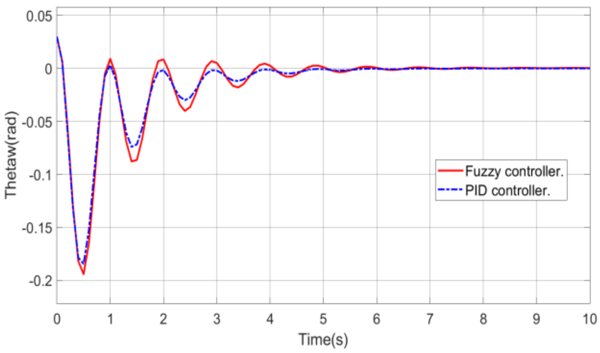

Fig. 11. Comparison of wheel angle proposed for Fuzzy controller and PID controller.

Tab. 1. Simulation results.

	e_{xl}	t_{xl} (s)
θ_b	0	7
θ_w	0	7

From simulation, the similarity between the two controllers are:

- Firstly, short settling time in both cases demonstrates the system's rapid stabilization under control. This suggests a commonality in how both controllers respond to the system's dynamic fluctuations.
- Secondly, oscillation characteristics such as frequency and amplitude are similar, highlighting uniformity in the oscillatory structure of system when both controllers are applied.

Simulation demonstrates that fuzzy controller trained by using the PID approach has met control requirements. The system has achieved stability at the equilibrium position.

5. Conclusion

In this paper, BoW has been considered. The mathematical model of the system has been developed to facilitate the simulation of the system. A PID controller has been designed to stabilize the system. The utilization of the Anfis tool for learning based on the PID controller is feasible, resulting in good control performance and close adherence to the PID controller. This opens up a possibility for designing intelligent controllers that learn from existing controllers in an easy manner.

Acknowledgement

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6. References

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