A COMPARISON OF CONTROL SCHEMES FOR UNDERACTUATED PENDUBOT SYSTEM

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Abstract: This paper presents a comparison of the three distinctive well-known control techniques, including Linear Quadratic Regulator (for short, LQR), Sliding Mode Control (for short, SMC), and Fuzzy Logic Control (for short, FLC). Each controller is in a different control category; LQR is an optimal controller employed in both linear and nonlinear systems, SMC is a nonlinear controller commonly applied in a nonlinear system, and the Fuzzy controller is the intelligent controller which can be archived by user's experiment or other learning techniques. Beside the three main controllers for stabilizing at the equilibrium point, the swing-up controller is designed based on energy-based method to bring the system at the initial position close to the equilibrium points. Finally, the performance and the validity of the three methods are verified through both simulation and experimental results.
Keywords: pendubot, linear quadratic regulator, input-output linearization, fuzzy logic control, comparison.

1. Introduction

Pendubot Robot, also known as Pendubot, is a classical under-actuated mechanical system which has the control inputs less than control actuators. The pendubot system is considered to have a vital role in scientific research, especially in control theory. Many techniques for controlling the pendubot system have been introduced in recent years, but only three of them will be presented in this paper - Linear Quadratic Regulator (LQR), Sliding Mode Controller(SMC), and Fuzzy controller.

In [1], Hung.V.P examined two controllers - PID and LQR combined with Genetic Algorithm (GA) to optimize the controller's parameters at an equilibrium point. The control technique for the pendubot system using Fuzzy logic was introduced in [2]. The authors in both papers [1] and [2] only had their research in simulation but not showing the experimental results. In [3], The results in simulation and experiments in controlling the pendubot system to stabilize at its equilibrium point, swing up from the initial points, and track the arbitrary wave, such as cosine wave, were shown. However, the author only deep-dived into one control technique, so it is hard to identify which technique gives a better response for the pendubot system. In [4], the authors designed the energy-based method to swing up and balance the system at the equilibrium points. The hierarchical sliding mode control method presented in [5] was proven its capability of controlling underactuated systems by the authors but only in simulation. In [6], the ANFIS was introduced to achieve the Fuzzy Logic rules to stabilize the system. In this paper, we present a comparison of the three control techniques, including LQR, SMC, and Fuzzy Control. The swing-up controller for the pendubot system is implemented by an energy-based method to bring the system at the initial unstable position close to the working points of the three controllers previously mentioned. Both simulation and experimental results are shown to evaluate.

This paper is organized in the following way. The dynamic mathematical equations of the Pendulum Robot system, including friction, are described in Section 2. In Section 3, the proposed control techniques are designed. Simulation and Experiment results are presented in Section 4. The conclusion is presented in Section 5.

2. Dynamic System

Physical structure of Pendubot is shown in Fig 1.

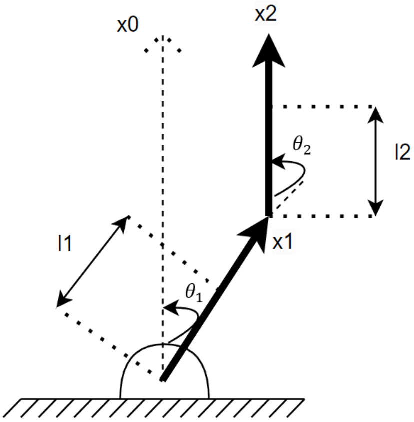

Fig. 1. Pendubot system.

The equations of motion in matrix form are presented as follows:

$$M(q)\ddot{q}+V(q,\dot{q})\dot{q}+G(q)=\tau \tag{1}$$

with τ is the vector of torque applied to the link, $q=\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$, $\dot{q}=\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$, and $\ddot{q}=\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}$ is the vector of joint motions and other parts are shown as follows

$$M(q)=\begin{bmatrix} \beta_1+\beta_2+2\beta_6\cos(q_2) & \beta_2+\beta_3\cos(q_2) \\ \beta_2+\beta_3\cos(q_2) & \beta_2 \end{bmatrix} \tag{2}$$

$$V(q,\dot{q})=\begin{bmatrix} -\beta_3\sin(q_2)\dot{q}_2 & -\beta_3\sin(q_2)\dot{q}_2-\beta_3\sin(q_2)\dot{q}_1 \\ \beta_3\sin(q_2)\dot{q}_1 & 0 \end{bmatrix} \tag{3}$$

$$G(q)=\begin{bmatrix} \beta_4 g\cos(q_1)+\beta_5\cos(q_1+q_2) \\ \beta_5\cos(q_1+q_2) \end{bmatrix} \tag{4}$$

$$\beta_1=m_1l_1^2+m_2l_1^2 \tag{5}$$

$$\beta_2=m_2l_{c2}^2 \tag{6}$$

$$\beta_3=m_2l_1l_2 \tag{7}$$

$$\beta_4=m_1l_1+m_2l_1 \tag{8}$$

$$\beta_5=m_2l_2g \tag{9}$$

$$\beta_6=m_1m_2 \tag{10}$$

The vector of torque applied is transformed to vector of voltage as given below:

$$\tau=a_1v-a_2\dot{q}_1-a_3\ddot{q}_1 \tag{11}$$

$$a_1=\frac{K_t}{R_m} \tag{12}$$

$$a_2=\frac{K_tK_b}{R_m} \tag{13}$$

$$a_3=J_m \tag{14}$$

The parameters of the Pendubot system are given as below:

Tab. 1 Parameters of Pendubot system

Parameter	Value	Unit	Description
m_1	0.16	kg	Mass of 1st link
l_1	0.2	m	Length of the 1st link
l_{1c}	0.1	m	Distance to center of mass (the 1st link)
I_1	0.00222	kgm^2	Inertia of the 1st link
m_2	0.066	kg	Mass of 2nd link
l_2	0.22	m	Length of the 2nd link
l_{2c}	0.11	m	Distance to center of mass (the 1st link)
I_2	0.00106	kgm^2	Inertia of the 1st link
K_t	0.0198	Nm	Motor Torque Constant
K_b	0.0198	V	Back EMF Constant
R_m	6.835	Ω	Armature Resistance
J_m	0.000134	kgm^2	Equivalent moment of inertia at the load
v	NA	V	Voltage applied to DC motor

3. Control Techniques

3.1. Energy-Based Method

Now, we denote x as q, linearizing the system around the equilibrium position as follows

$$x=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}=\begin{bmatrix} q_1 & \dot{q}_1 & q_2 & \dot{q}_2 \end{bmatrix}$$

The initial position of x is

$$x_{init}=\begin{bmatrix} x_{1\_init} & x_{2\_init} & x_{3\_init} & x_{4\_init} \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

Where

x_1 and x_3 are the angular position of link 1 and link 2, respectively.

x_2 and x_4 are angular velocity of link 1 and link 2, respectively.

The newly formed equations of system from (2),(3), and (4) are substituted into (1) to achive (15),(16),(17), and (18).

$$\dot{x}_1=x_2 \tag{15}$$

$$\dot{x}_2=f_1(x)+b_1(x)u \tag{16}$$

$$\dot{x}_3=x_4 \tag{17}$$

$$\dot{x}_4=f_2(x)+b_2(x)u \tag{18}$$

with $f_1(x)$, $b_1(x)$, $f_2(x)$, $b_2(x)$, and u are presented in the Table 2 below.

Tab. 2 Parts of the newly formed system.

$\dot{x}_1 = \dot{q}_1$
$\dot{x}_2 = \ddot{q}_1$
$\dot{x}_3 = \dot{q}_2$
$\dot{x}_4 = \ddot{q}_2$
$f_1(x) = \dfrac{\left(\beta_3^2 x_2^2 \sin(x_1 - x_3)\cos(x_1 - x_3) - \beta_3\beta_5 \cos(x_1 - x_3)\sin x_1 - \beta_2 a_2 x_2 + \beta_2\beta_4 g \sin x_1 - \beta_2\beta_3 x_4^2 \sin(x_1 - x_3)\right)}{\beta_2\left(\beta_1 + a_3 - 2\cos(x_1 - x_3)(\beta_3 - \beta_6)\right) - \beta_3^2 \cos(x_1 - x_3)^2}$
$b_1(x) = \dfrac{a_1\beta_2}{\beta_2\left(\beta_1 + a_3 - 2\cos(x_1 - x_3)(\beta_3 - \beta_6)\right) - \beta_3^2 \cos(x_1 - x_3)^2}$
$f_2(x) = A_1(x) * A_2(x) - A_3(x) * A_4(x)$ *Where* $A_1(x) = \dfrac{1}{a_3\beta_2 - \beta_1\beta_2 + \beta_3^2 \cos^2(x_1 - x_3)}$; $A_2(x) = \left[-\beta_2 a_2 x_2 + \beta_3^2 x_2^2 \cos(x_1 - x_3)\sin(x_1 - x_3) - \beta_2\beta_4 g \cos\left(x_1 - \frac{\pi}{2}\right) + \beta_2\beta_3 x_4^2 \sin(x_1 - x_3) + \beta_3\beta_5 g \cos(x_1 - x_3)\cos\left(x_3 - \frac{\pi}{2}\right)\right]$ $A_3(x) = \dfrac{1}{\beta_3^2 \cos^2(x_1 - x_3) - \beta_1\beta_2}$ $A_4(x) = \left[A_5(x) - A_6(x) + A_7(x) + A_8(x)\right]$ *Where* $A_5(x) = \beta_5 g \sin x_3 \left(\beta_1 + \beta_3 \cos(x_1 - x_3)\right)$ $A_6(x) = \left[\beta_5\left(\beta_2 + \beta_3\cos(x_1 - x_3)\right)\left(a_2 x_2 + \dfrac{a_3}{a_3\beta_3 - \beta_1\beta_2 + \beta_3^2\cos^2(x_1 - x_3)} * \left(\begin{array}{l} -\beta_2 a_2 x_2 + \beta_3^2 x_2^2 \cos(x_1 - x_3)\sin(x_1 - x_3) + \\ -\beta_2\beta_4 g \cos\left(x_1 - \frac{\pi}{2}\right) + \beta_2\beta_3 x_4^2 \sin(x_1 - x_3) + \\ +\beta_3\beta_5 g \cos(x_1 - x_3)\cos\left(x_3 - \frac{\pi}{2}\right) \end{array}\right) + \beta_4 g \sin x_1\right)\right]$ $A_7(x) = \beta_3 x_2^2 \sin(x_1 - x_3)\left(\beta_1 + \beta_3 \cos(x_1 - x_3)\right)$ $A_8(x) = \beta_3 x_4^2 \sin(x_1 - x_3)\left(\beta_2 + \beta_3 \cos(x_1 - x_3)\right)$
$b_2(x) = \dfrac{\beta_2 a_1}{a_3\beta_2 - \beta_1\beta_2 + \beta_3^2\cos^2(x_1 - x_3)} - \dfrac{a_1 + \beta_2 a_1 a_3}{\beta_3^2\cos^2(x_1 - x_3) - \beta_1\beta_2}$
$u = \tau$

The Lyapunov function for the energy-based method :

$$V = \frac{1}{2}K_e(E - E_0)^2 + \frac{1}{2}K_p x_1^2 + \frac{1}{2}K_d x_3^2 \quad (19)$$

$$\dot{V} = K_e(E - E_0)\dot{E} + K_p x_1 x_2 + K_d x_2 \dot{x}_2 \quad (20)$$

With K_e, K_p, and K_d are the coefficients of the Energy-based controller.

We have the energy equation and its derivation as follows

$$E=\frac{1}{2}x_2^2(m_2l_2^2+m_2l_1^2+2m_2l_1l_2\cos(x_2)+I_2+I_1) \quad (21)$$
$$+x_2x_4(m_2l_2^2+I_2+m_2l_1l_2\cos(x_2))$$
$$+\frac{1}{2}x_4^2(m_2l_2^2+I_2)+m_1gl_1\cos(x_1)$$
$$+m_2l_2g\cos(x_1+x_2)+m_2gl_1\cos(x_1)$$

$$\dot{E}=-b_1x_2^2-b_2x_4^2+x_2u \quad (22)$$

The desired energy at zero presented as follows

$$E_0=m_1gl_1+m_2gl_2+m_2gl_1 \quad (23)$$

We have the control rule as follow

$$u=\frac{b_1K_e(E-E_0)-K_px_1-K_df_1-K_1x_2}{K_e(E-E_0)+K_db_1(x)} \quad (24)$$

If the input u is applied to the system, the derivative Lyapunov function is negative in (30)

$$\dot{V}=-k_1x_2^2 \quad (25)$$

The control rule above ensures the system asymptotically stablize at the equilibrium points arcoding to [4]. The parameters are chosen by trial and error.

3.2. Linear Quadratic Regulator (LQR)

The state space representation of Pendubot is presented as follows:

$$\dot{x}=Ax+Bu \quad (26)$$

With

$$A=\begin{bmatrix}\frac{\partial\dot{x}_1}{\partial x_1}&\frac{\partial\dot{x}_1}{\partial x_2}&\frac{\partial\dot{x}_1}{\partial x_3}&\frac{\partial\dot{x}_1}{\partial x_4}\\ \frac{\partial\dot{x}_2}{\partial x_1}&\frac{\partial\dot{x}_2}{\partial x_2}&\frac{\partial\dot{x}_2}{\partial x_3}&\frac{\partial\dot{x}_2}{\partial x_4}\\ \frac{\partial\dot{x}_3}{\partial x_1}&\frac{\partial\dot{x}_3}{\partial x_2}&\frac{\partial\dot{x}_3}{\partial x_3}&\frac{\partial\dot{x}_3}{\partial x_4}\\ \frac{\partial\dot{x}_4}{\partial x_1}&\frac{\partial\dot{x}_4}{\partial x_2}&\frac{\partial\dot{x}_4}{\partial x_3}&\frac{\partial\dot{x}_4}{\partial x_4}\end{bmatrix} \quad (27)$$

$$B=\begin{bmatrix}\frac{\partial\dot{x}_1}{\partial u}\\ \frac{\partial\dot{x}_2}{\partial u}\\ \frac{\partial\dot{x}_3}{\partial u}\\ \frac{\partial\dot{x}_4}{\partial u}\end{bmatrix} \quad (28)$$

Following the value in Tab. 1, we have value of A and B matrices as given below:

$$A=\begin{bmatrix}0&1&0&0\\ 54.2681&-11.3475&-62.6168&0\\ 0&0&0&1\\ -51.7634&13.9007&91.4205&0\end{bmatrix} \quad (29)$$

$$B=\begin{bmatrix}0\\ 113.4750\\ 0\\ -139.0068\end{bmatrix} \quad (30)$$

$$Q=\begin{bmatrix}1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1\end{bmatrix} \quad (31)$$

$$R=1 \quad (32)$$

By solving the Ricaccti equations as follow

$$A^TS+SA-SBR^{-1}B^TS+Q=0 \quad (33)$$

We , finally, achieve the final result for the gain K.

$$K=[-70.44 -20.555 -69.72 -18.05] \quad (34)$$

3.3 Sliding Mode control

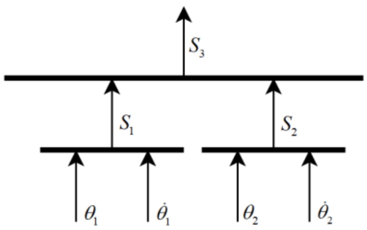

Fig. 2. The structure of the hierachical SMC.

The sliding mode surfaces are chosen arcoding to [5], or the structure in Fig.2, we have those as follows

$$S_1=K_{1\_smc}x_1+x_2 \quad (35)$$

$$S_2=K_{2\_smc}x_3+x_4 \quad (36)$$

$$S_3=K_{3\_smc}S_1+S_2 \quad (37)$$

With K_{1_smc} , K_{2_smc} ,and K_{3_smc} are the coefficients of the SMC controller.

To achieve the final input torque to the system , we first applied the Lyapunov technique

$$V=\frac{1}{2}S_3^2 \quad (38)$$

Next, we take derivative V

$$\dot{V}=S_3\dot{S}_3 \quad (39)$$
$$=K_{3\_smc}K_1x_2+K_{2\_smc}x_4+K_{3\_smc}\dot{x}_2+\dot{x}_4$$

To make sure that all the sliding surface reach zero, we must have $\dot{V}(t) < 0$ when t approaches to infinity, the $\dot{S}_3$

$$\dot{S}_3 = -\delta sign(S_3) \qquad (40)$$

With δ is the coefficient of the SMC controller.

From (16),(18),(19),(20),(21), (22), and (45) we achieve that final input as follows

$$u = \frac{-K_{3\_smc}K_{1\_smc}x_2 - K_{2\_smc}x_4 - K_{3\_smc}f_1(x) - f_2(x) - \delta sign(S_3)}{K_{3\_smc}b_1(x) + b_2(x)} \qquad (41)$$

The input (40) makes sure the derivative of V always be less than zero if all the coefficients are larger than zero.

$$\dot{V} = S_3\dot{S}_3 = -\delta|S_3| < 0 \qquad (42)$$

3.4. Fuzzy Control

3.4.1. Adaptive Neuro Fuzzy Inference System

To achieve the Fuzzy rules, in this paper, we applied the ANFIS (Adaptive neuro fuzzy inference system) technique in MATLAB software. [6]

Parameters chosen for the training batch :

- Membership function : 3
- Number of input mfs : [4 3 4 3]
- Membership type : Gauss
- Number of epoch : 8
- Optimization method : Hybrid

The data used for the training are achieved through the LQR control in (29),(30),(31),(32).

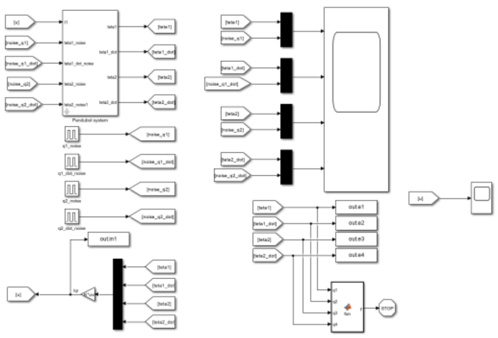

Fig .3 The training program in Simulink.

3.4.2. The training results

Tab. 3. The training results.

```
Designated epoch number reached. ANFIS training completed at epoch
2.
Minimal training RMSE = 8.49123e-05
ANFIS info:
        Number of nodes: 323
        Number of linear parameters: 144
        Number of nonlinear parameters: 42
        Total number of parameters: 186
        Number of training data pairs: 20001
        Number of checking data pairs: 0
        Number of fuzzy rules: 144
Start training ANFIS ...
1        8.49123e-05
2        0.00317121
Designated epoch number reached. ANFIS training completed at epoch
2.
Minimal training RMSE = 8.49123e-05
```

The result shown in Tab. 3 means that the Fuzzy controller controlling the pendubot system had the same value outputs as the LQR did to the system.

4. Final Results

The swing-up control from (24) brings the system to the equilibrium point with the choices of coefficients as follows $K_1 = 33$, $K_e = 25.5$, $K_p = 87$, $K_d = 8$, and then switches to one of the three controllers, SMC, LQR, and Fuzzy, to stabilize.

The coefficients of the SMC are chosen as follows $K_{1\_smc} = 8$, $K_{2\_smc} = 8.5$, $K_{3\_smc} = 1.17$, and $\delta = 6.57$

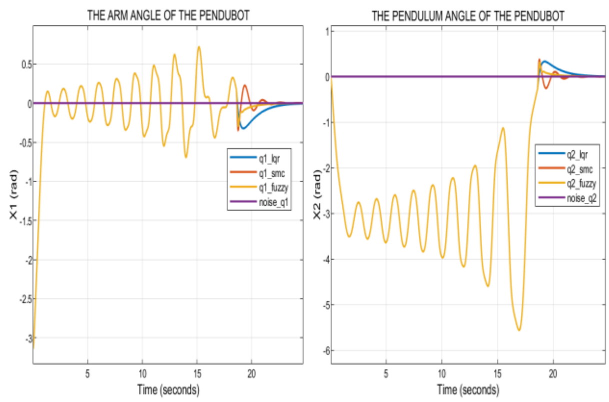

Fig. 4. The system response the swing-up controller.

The energy-based method makes sure the 1st pendulum reaching to its equilibrium first, then swings the 1st pendulum to create torque for the 2nd pendulum to gradually increase its energy until it reaches the desired energy value as in (23) and then switches to one of the three controllers , SMC,LQR, and Fuzzy at the 18th seconds in Fig.4. Intuitively, the transient time for the pendubot using the energy-based method is approximately 17 seconds due to the choices of the coefficients, it could be better if we do more tests to find the best coefficients or use some searching methods such as Genetic Algorithm.

It also includes the signal of stabilizing the system of the three controllers, but it will be shown more clear in Fig.5.

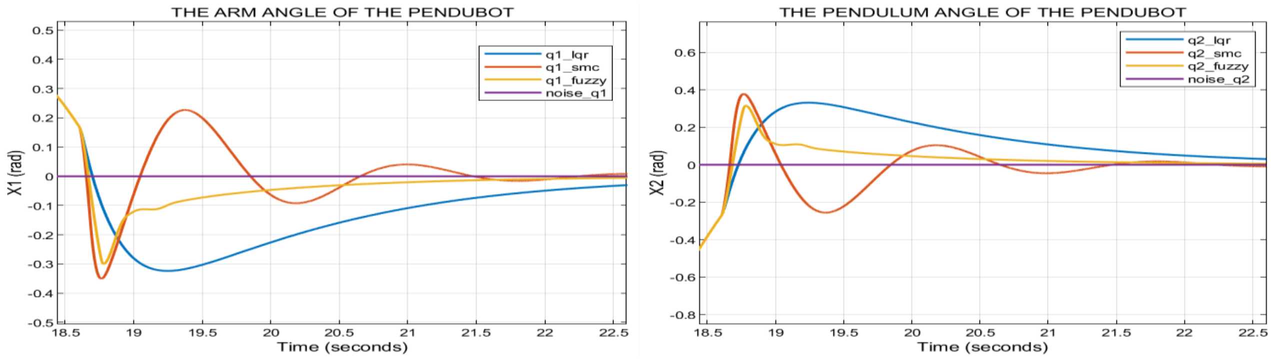

Fig. 5. The response of each controller.

When we zoom in between 18.5 seconds to 22.5 seconds, we can see each transient of the three controllers. As no noise is applied to the system, the amplitude of the fuzzy controller seems smaller or better than others. The three proposed controllers have the capability to stable at the equilibrium point.

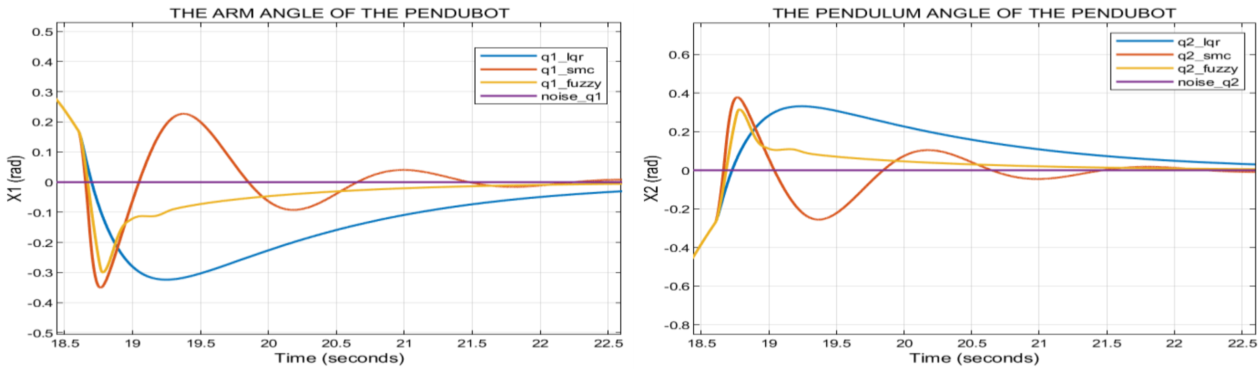

Fig. 6. The system response when having noise.

With the same scale, when noises are applied to the system, the Fuzzy controller at first still can hold the system to the equilibrium point but cannot hold forever. The SMC shows its robustness to the noise applied to the system. The LQR also performs a good quality but as the result shows that it is not good as the SMC, this could possibly be the choice of weigh matrix Q and R are not optimized enough for the system.

5. Conclusions

We have presented the control strategies for the pendubot that first swings it up close to the equilibrium point and then applies one of the three controllers: SMC, LQR, or Fuzzy, to maintain stability at its desired point. Moreover, we can observe the robustness of each controller when having noises or not. Lastly, the parameters of each controller can either choose by experiment or a searching algorithm such as Genetic Algorithm to optimize the robustness of each controller.

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7. References

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