GA-PID CONTROL FOR BALL AND BEAM: SIMULATION AND EXPERIMENT

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Abstract: The Ball and Beam system with Deviated Axis is a single input-muti output (SIMO) system commonly used in laboratories to test control algorithms. In this paper, we build and investigate an PID-GA controller in simulation and apply to real model. The controller demonstrates the ability to control the balancing statement in different desired positions. Next, we conduct a survey of the above method in the object name Ball and beam system with deviated axis through STM32F4. Through simulation and experiment, our PID controller has successfully controlled the system and GA-PID has optimize well PID parameters. In addition, the control parameters had been adjusted to verify and summerize the theoretical rules.
Keywords: PID control, Ball and Beam, genetic algorithm, STM32F4, SIMO system.

1. Introduction

The ball and beam system (B&B) [1] is a familiar balancing system and is used in universities frequently. Therefore, Quanser company invented a standard model [2]. Due to this platform, many experiments are operated to test algorithms [3]. Sliding control [8] operates well on this system. But, chattering phenomenon is a big problem of this method. Moreover, the high nonlinear functions in sliding algorithms make it difficult to be widely used. Thence, LQR [9], fuzzy [11], neural controllers [12] are utilized to overcome this phenomenon. However, these methods are just focused in laboratory and academy. Among methods, PID controller is still the most popular algorithm in both industry and academy [4]. Thence, it should still be studied more for training students in laboratory. However, PID is not guaranteed by mathematics. Thence, the calibration of PID must be done by trial-and-error test. This action takes time. Thence, genetic algorithm (GA) is used to optimize the control parameters [6]. In this paper, GA is proved to optimize successfully PID controller through generations, in both simulation and experiment. In order to self-build an experimental model which imitates Quanser model for laboratory of our university, we present a real model of B&B which is cheaper than Quanser's model. Due to this model, we design PID control and test this algorithm on both simulation and experiment. Thence, through this research, we confirm an idea that B&B can be self-built as cheap and suitable model for laboratory to test, at least, GA and PID for training and researching. It can be popularize for universities in Vietnam for training algoritghms.

2. Mathematical Equations

Mathematical structure of classical B&B is shown in Fig. 1. Rotation of motor

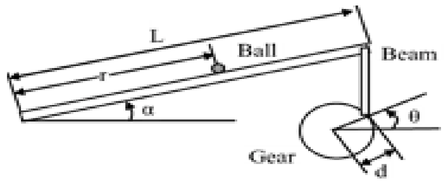

Fig. 1. Mathematical model of B&B [5]

As shown in Fig.1. the system has metal balls placed on a pair of parallel bars called beam so that the balls can roll along the length of the rod. A swing arm is mounted at the end of the beam and is attached to the gearbox. When the gearbox rotates an angle θ, it changes the angle of the beam (denoted by α) respect to the horizontal. Then, Under the gravity force make the ball roll along the bar in the positive or negative direction depending on the value α .

From [5], dynamic equations of B&B are:

$$\ddot{\alpha} = \frac{-(2m_b r\dot{r}\dot{\alpha} + \frac{L}{2} g m_B \cos(\alpha) + \mathrm{g m}_b \mathrm{r}\cos(\alpha)) + u}{m_b r^2 + Jb} \quad (1)$$

$$\ddot{r} = \frac{m_b r\dot{\alpha}^2 - m_b g \sin(\alpha)}{m_b + \frac{J_B}{R_B{}^2}} \quad (2)$$

where system parameters and variables are shown in Tab. 1 below. System parameters are measured from real model in Fig. 16.

The relation of angle (α) and angle (θ) is

$$\sin(\alpha) = \frac{d}{L}\sin(\theta) \quad (3)$$

To apply the equation to the real model, the moment of motor in equations (1), (2) needs to be converted into the voltage input. From [7], with relation of voltage and moment, we convert (1) and (2) into (4) and (5) below

$$\ddot{\alpha} = \frac{-\left[\begin{array}{l} 2T_f k_3 - 2ek_1k_3 + 2\dot{\alpha}k_2k_3^2 + 4\dot{\alpha}m_B r\dot{r} + \\ +Lgm_b cos(\alpha) + 2gm_B rcos(\alpha) + 2C_m\dot{\alpha}k_3^2 \end{array}\right]}{(2(m_B r^2 + J_b + J_m k_3^2))} \quad (4)$$

$$\ddot{r} = \frac{-(R_B{}^2(-m_B r\dot{\alpha}^2 + gm_B sin(\alpha)))}{(m_B R_B{}^2 + J_B)} \quad (5)$$

Tab. 1. System parameters and variables of B&B

Symbols	Meaning	Value Measurement
$r(t)$	Ball position	
$\alpha(t)$	Beam Angle	
$u(t)$	Torque apply to beam	
d	Lever Radius	0.075 (m)
g	Acceleration due to gravity	9.81(m/s^2)
m_B	Ball mass	0.065 (kg)
m_b	Beam mass	0.34 (kg)
L	Beam length	0.54 (m)
d_t	Gear ratio	5.6
J_B	moment of inertia of the ball	$\frac{2}{5}m_B R_B{}^2$ $(kg.m^2)$
J_b	moment of inertia of the beam	$\frac{1}{3}m_b L^2$ $(kg.m^2)$
R_B	Ball radius	0.0125 (m)
R_m	Motor resistance	6.83527 (Ω)
K_t	Torque constant	0.064943(N.m/A)
K_b	Back EMF constant	0.064943(V.s/rad)
C_m	Coefficient of viscous friction	0.00034(N.m/(rad/s))
J_m	Moment of inertia of Rotor	0.000134(kg.m^2)
T_f	Moment friction	0.010764 (N.m)
k_1	Constant	$\frac{K_t}{R_m}$
k_2	Constant	$\frac{K_t K_b}{R_m}$
k_3	Constant	$\frac{L.d_t}{d}$
e	Voltage Input	(V)

3. Simulation

Structure of simulation program of controlling B&B is shown in Fig. 2. Explanation of blocks in this figure is listed in Tab. 2.

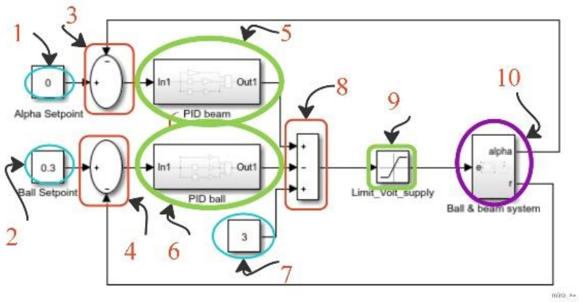

Fig. 2. Matlab PID simulation program

Tab. 2. Blocks in simulation program

(1)-	Beam position Setpoint (rad)
(2)-	Ball position Setpoint (m)
(3)-	Take beam angle error compare to setpoint
(4)-	Take ball position error compare to setpoint
(5)-	PID Beam
(6)-	PID Ball
(7)-	Volt addition for keep beam angle
(8)-	Sum of PIDs
(9)-	Limit voltage supply
(10)-	Ball & beam system mathematical (Fig. 3)

Ball initial position is always higher than set-point so that the direction should be negative,

Initial value of angle of beam is a negative one trying to reach set-point 0 so the direction is positive.

The 3V source addition is applied to prevent the beam position lower down cause by the gravity.

The B&B block is shown in Fig. 3, the explanation in detail of Fig. 3 is listed in Tab. 3

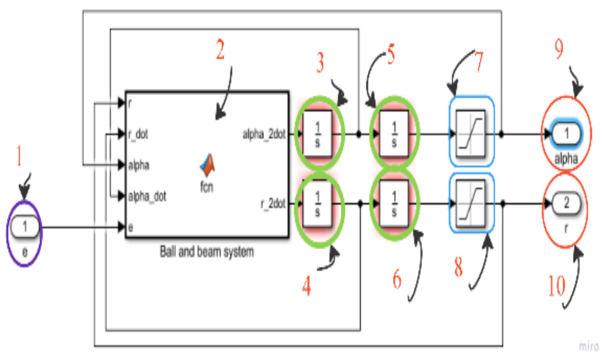

Fig. 3. B&B block

Tab. 3. Blocks in simulation of B&B block

(1)-	Voltage input (v)
(2)-	Mathematical of system
(3)-	Integrator
(4)-	Integrator
(5)-	Integrator
(6)-	Integrator
(7)-	Limit the Beam Angle
(8)-	Limit Ball Position
(9)-	Beam Angle (rad)
(10)-	Ball position (m)

Based on equations (4), (5), Fig. 2 and Fig. 3, a simulation system is created. Some steps are made to convert the initial equation to desired value r, alpha. But friction, starting velocity.... are neglected.

Simulation results are shown from Fig. 4 toFig. 10 below. In simulation and experiment, we just focus on ball position. The angle of beam is neglected because when ball is balanced, the angle is at horizontal place. Therefore, motion of ball can present quality of controller.

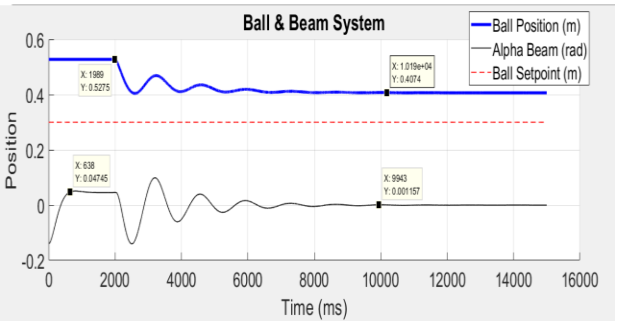

Fig. 4. Kp_ball=10.9; Kd_ball=16.9; Kp_beam=24.6; Kd_beam=2.1

State steady error	$0.1074(m)$
Settling time	$10.19(s)$
Undershoot:	0%

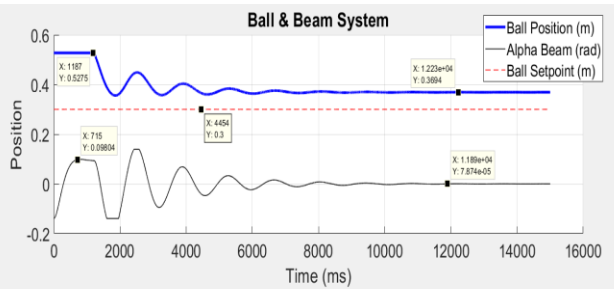

Fig. 5. Kp_ball=15.9; Ki_ball=0; Kd_ball=16.9; Kp_beam=24.6; Ki_beam=0; Kd_beam=2.1

State steady error	$0.0694(m)$
Settling time	$12.23(s)$
Undershoot:	0%

From Fig. 4 and Fig. 5, increasing Kp_ball can reduce state steady error from 0.1074(m) down to 0.0694(m). In constract, under that calibration, settling time raise from 10.19s to 12.23s.

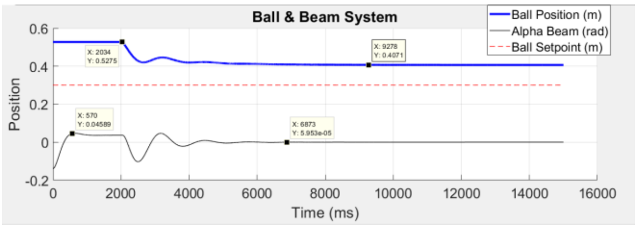

Fig. 6. Kp_ball=10.9; Ki_ball=0; Kd_ball=16.9; Kp_beam=30.6; Ki_beam=0; Kd_beam=2.1

State steady error	$0.1071(m)$
Settling time	$9.278(s)$
Undershoot:	0%

From Fig. 4 and Fig. 6, increasing Kp_beam effects happen on alpha response likewise ball response, help reducing the settling time from 10.19(s) down to 9.278(s) and a little bit decline on error about 0.0003(m)

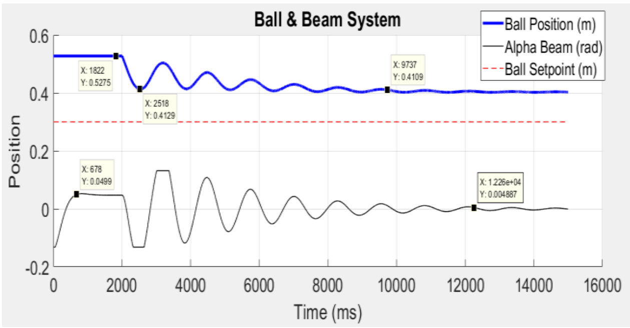

Fig. 7. Kp_ball=10.9; Ki_ball=0; Kd_ball=20.9; Kp_beam=24.6; Ki_beam=0; Kd_beam=2.1

State steady error	$0.1109(m)$
Settling time	$9.737(s)$
Undershoot:	0%

From Fig. 4 and Fig. 7, raising Kd_ball from 16.9 to 20, we reduce settling time from 10.19s to 9.737s.

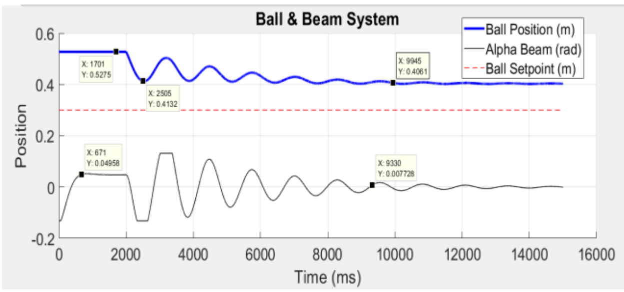

Fig. 8. Kp_ball=10.9; Ki_ball=0; Kd_ball=16.9; Kp_beam=24.6; Ki_beam=0; Kd_beam=5.1

State steady error	$0.1061(m)$
Settling time	$9.945(s)$
Undershoot:	0%

From Fig. 4 and Fig. 8, changing Kd_beam from 2.1 to 5 helps reducing settling time, small changes on state error.

From Fig. 4, Fig. 5, Fig. 6, Fig. 7, Fig. 8, adjusting Kp, Kd is recommended. We keep Ki with value 0. Now, we put a small value to Ki to verify the rule.

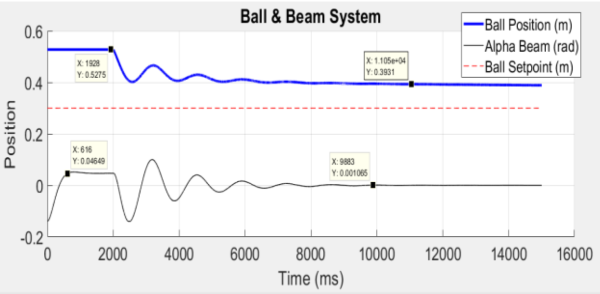

Fig. 9. Kp_ball=10.9; Ki_ball=0.1; Kd_ball=16.9; Kp_beam=24.6; Ki_beam=0.1; Kd_beam=2.1

State steady error	$0.0931(m)$
Settling time	$11.05(s)$
Undershoot:	0%

State steady error redeuce from 0.1074(m) (in Fig. 4) to 0.0931(m) (in Fig. 9). Error had been reducing over the time but Ki very small take much time to reach the setpoint and settling state.

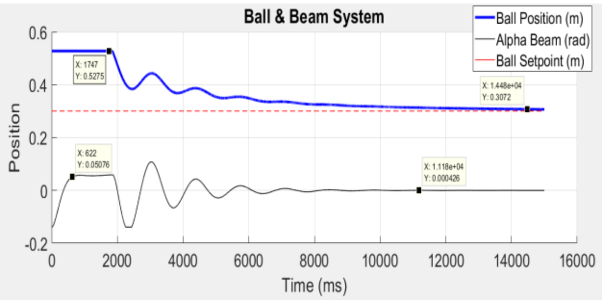

Fig. 10. Kp_ball=10.9; Ki_ball=1; Kd_ball=16.9; Kp_beam=24.6; Ki_beam=1; Kd_beam=2.1

State steady error	$0.0072(m)$
Settling time	$14.48(s)$
Undershoot:	0%

From Fig. 9 and Fig. 10**,** error reduces from 0.0931m to 0.0072m.

The PID rules calibration proved that with higher Kp correspond to lower state steady error. Increasing Ki makes the system reduce a large amount of error. Higher Kd helps the plant to detect the small trending and make the fast response. Based on that knowledgement, some goals will be set, such as, reaching the set point with the shortest time or no overshoot, undershoot or smallest error. When GA are applied, we can use this method to change result with different desires. Moreover, finding PID parametters manually takes much time and efforts. Then, GA is a solution (Fig. 11).

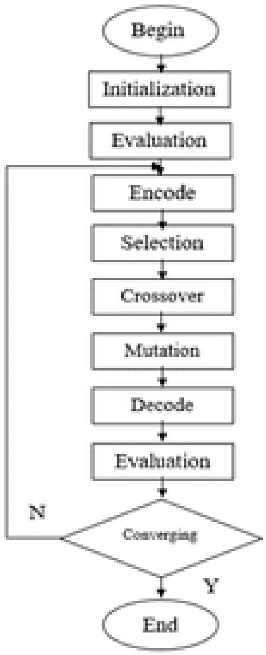

Fig. 11. GA flowchart

From **Fig. 12** to **Fig. 15** Simulation result are represented to illustrate the theory that next generation are better. Fitness function is chosen as

$$J\min = \sum_{k=1}^{N}\left[e^2(k) + \alpha^2(k)\right] \tag{6}$$

where e(k) is error at sample k of ball position and set-point; $\alpha(k)$ is value of angle beam at sample k.

By (6), the better result is guaranteed by smaller value of Jmin.

Tab. 4. generation #8 of maximum 20000

Kp_ball	43.7651
Ki_ball	0
Kd_ball	21.8410
Kp_beam	80.7536
Ki_beam	0
Kd_beam	3.3305
Jmin	222.9637

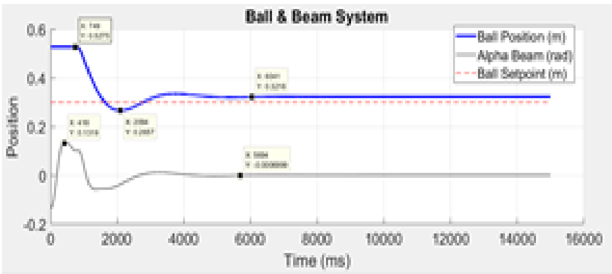

Fig. 12. Kp_ball=43.7651; Kd_ball=21.8410; Kp_beam=80.7536; Kd_beam=3.3305; Jmin=222.9637

State steady error	$0.0216(m)$
Settling time	$6.041(s)$
Undershoot:	17.38%

Tab. 5. generation #9 of maximum 20000

Kp_ball	59.7654
Ki_ball	0
Kd_ball	22.8796
Kp_beam	88.7535
Ki_beam	0
Kd_beam	3.3315
Jmin	181.8753

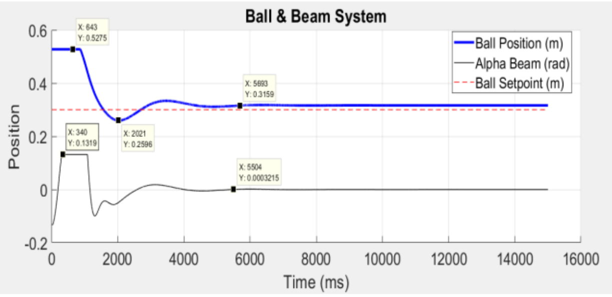

Fig. 13. Kp_ball 59.7654;Kd_ball 22.8796;Kp_beam 88.7535;Kd_beam 3.3315; Jmin 181.8753

State steady error	0.0159(*m*)
Settling time	5.693(*s*)
Undershoot:	17.82%

From Fig. 12 and Fig. 13, the later generation give better results than the older one. Error reduces from 0.0216(m) (in Fig. 12) to 0.0159(m) (in Fig. 13). Settling time 6.041s (in Fig. 12) down to 5.693s (in Fig. 13).

Tab. 6. generation #3 of maximum 20000

Kp_ball	83.6921
Ki_ball	5.9300e-04
Kd_ball	59.5891
Kp_beam	97.1635
Ki_beam	0.0065
Kd_beam	23.9235
Jmin	164.8814

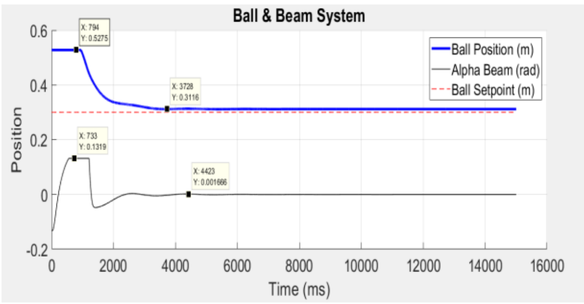

Fig. 14. Kp_ball=83.6921; Ki_ball=0.0006; Kd_ball=59.5891; Kp_beam=97.1635; Ki_beam=0.0065; Kd_beam=23.9235; Jmin=164.8814

State steady error	0.0118(*m*)
Settling time	3.728(*s*)
Undershoot:	0%

Settling time decreases from 5.693(s) in Fig. 13 to 3.728(s) in Fig. 14

Tab. 7. generation #65 of maximum 20000

Kp_ball	76.5369
Ki_ball	0.0919
Kd_ball	57.9051
Kp_beam	99.8906
Ki_beam	0.0449
Kd_beam	13.2738
Jmin	156.1760

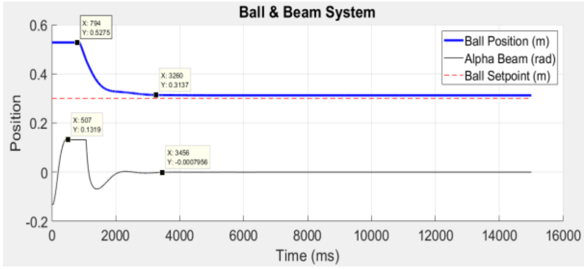

Fig. 15. Kp_ball 76.5369;Ki_ball 0.0919;Kd_ball 57.9051;Kp_beam 99.8906;Ki_beam 0.0449;Kd_beam 13.2738; Jmin 156.1760

State steady error	0.0137(*m*)
Settling time	3.26(*s*)
Undershoot:	0%

The next generation 65 compare with the previous generation 3, result show that settling time decrease from 3.726s (in Fig. 14) to 3.26s (in Fig. 15)

4. Experiment

4.1. Real Model

An experimental model is built as in Fig. 16. In this model, we use STMF407 as main processor due to its high speed. It can be embedded by Matlab/Simulink. Then, it is suitable to be a model for training in univeristy.

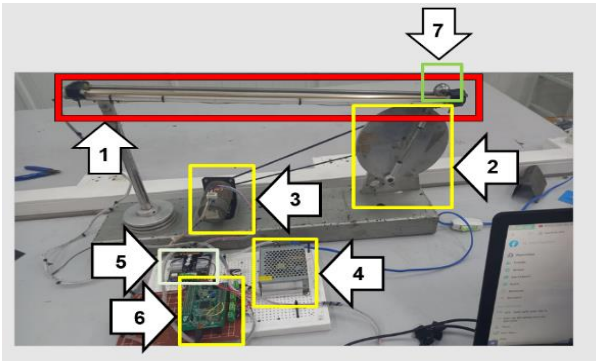

Fig. 16. Real research model

(1)- Beam
(2)- Level arm
(3)- Motor
(4)- 12V Source
(5)- H- Bridge Driver
(6)- STM32f407

4.2. Experimental Results

In experiment, we operate model with the parameters found by searching manually. Then, we verify the results by comparing GA-PID parameters and manual PID. The PID parameters shown in **Fig. 17** are found by seaching manually. Although it takes a long time to stabilize system at a fixed position, quality is still not good because long settling time is approximately 7,589s to reach equilibrium point of ball position. In addition, the problem in the state error when the error is up to 0.0541(m) which can be considered as a rather large one.

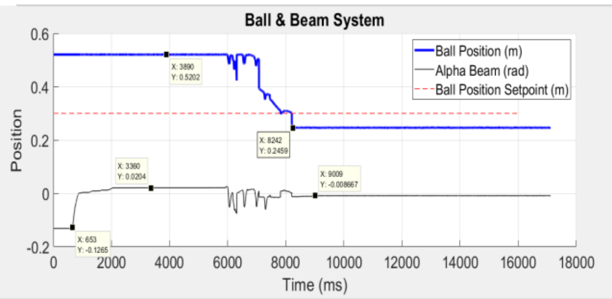

Fig. 17. Kp_ball=10.9; Kd_ball=16.9; Kp_beam=24.6; Kd_beam=2.1

State steady error	0.0541(m)
Settling time	7.589(s)
Undershoot:	0%

In Fig. 18, responses of experimental B&B under PID paremeters, which are optimiozed in simulation by GA in Tab. 4, are compared to manually calibrated PID in Fig. 17. The state steady error has been improved when down from 0.0541(m) (in Fig. 17) to 0.0371(m) (in Fig. 18). The graph show the better result not only on settling error but also on settling time. This time is reduced from 7.589(s) to just only 2.544(s).

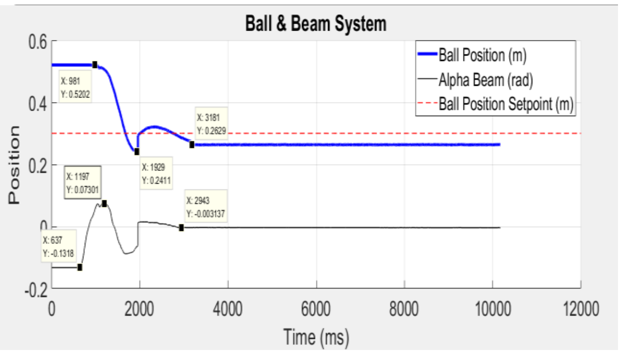

Fig. 18. Kp_ball=43.7651; Kd_ball=21.8410; Kp_beam=80.7536; Kd_beam=3.3305; Jmin=222.9637

State steady error	0.0371(m)
Settling time	2.544(s)
Undershoot:	9.0419%

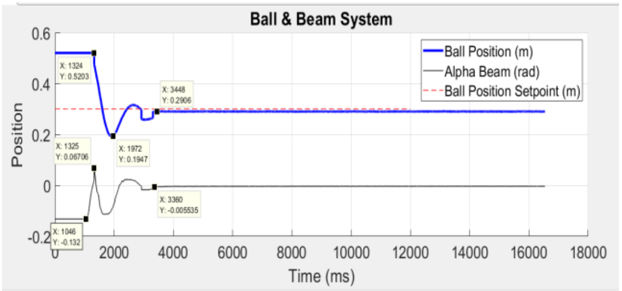

Fig. 19. Kp_ball=59.7654; Kd_ball=22.8796; Kp_beam=88.7535; Kd_beam=3.3315; Jmin=181.8753

State steady error	0.0094(m)
Settling time	2.402(s)
Undershoot:	33%

PID parameters are taken from the Tab. 5 (simulation) and applied in real model. The data received also proves that the later generation in Fig. 18 gives better results than former generation in Fig. 19. Then, optimization of GA is proved on both simulation and experiment.

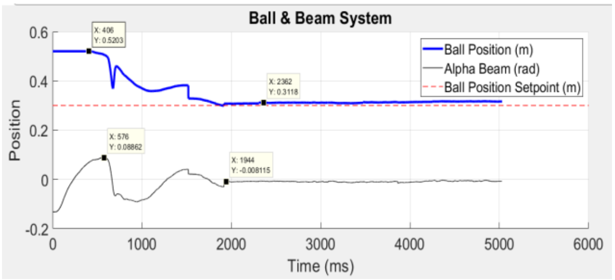

Fig. 20. Kp_ball=83.6921; Ki_ball=0.0006; Kd_ball=59.5891; Kp_beam=97.1635; Ki_beam=0.0065; Kd_beam=3.9235; Jmin=164.8814

State steady error	0.0118(m)
Settling time	2.362(s)
Undershoot:	0%

From the Fig. 17 to Fig. 19, the result is already good without Ki. But, for better response, Ki has been put and searched for appropriate value. In the Fig. 21 and Fig. 22, Ki can be free chosen. But in real model with the appearance of random noise, Ki should be limited for not created the unwanted output.

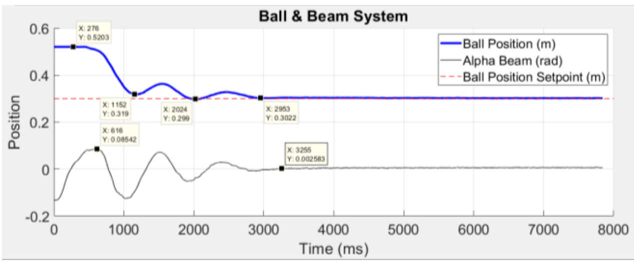

Fig. 21. Kp_ball=76.5369; Ki_ball=0.0919; Kd_ball=57.9051; Kp_beam=99.8906; Ki_beam=0.0449; Kd_beam=13.2738; Jmin=156.1760

State steady error	0.0022(m)
Settling time	2.953(s)
Undershoot:	1.0589%

Comparing response in Fig. 19 to it in Fig. 20, the undershoot had been reducing for a large amount. Using GA to find the different PID for different set point.Change set point is 0.2m, we have system response as in Fig. 22.

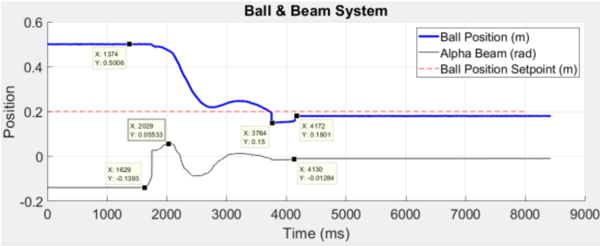

Fig. 22. Kp_ball=32.2484; Ki_ball=0; Kd_ball=24.8459; Kp_beam=57.1180; Ki_beam=0; Kd_beam=5.1547; Jmin=271.0209

State steady error	0.0199(m)
Settling time	2.543(s)
Undershoot:	16.71%

In Fig. 22, experimental results also prove that GA can optimize PID parameter for other specific operating point (0.2m) instead former set-point in Fig. 17, Fig. 18, Fig. 19, Fig. 20, Fig. 21 (0.2(m)).

5. Conclusion

In this paper, we use successfully PID controller. Rules of calibration of this method are presented. We also use GA to optimize PID controllers. And, GA is proved to help us find better controllers through generations. All results are tested on both simulation and experiment. Then, our experimental model is tested well by PID controllers with same parameters in simulation. So, it is suitable for more experimental studies in other controlling methods to verify theoretical rules.

6. References

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