A METHOD OF FUZZY ALGORITHM IN CONTROLLING BALL AND BEAM THROUGH SIMULATION AND EXPERIMENT

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Abstract: In this paper, we present a way to create fuzzy controller from LQR method by using ANFIS toolbox of Matlab. First, after proving the ability of stability of this SIMO system under LQR method in Matlab/Simulink, we create a fuzzy controller through ANFIS toolbox of Matlab. The data, which is used to train, is collected from responses of system under LQR controller. Also, we present a hardware platform of ball and beam system. Under this fuzzy controller, control quality of ball and beam is better than under LQR controller in both simulation and experiment.
Keywords: fuzzy, LQR, ball and beam, SIMO system.

1. Introduction

Ball and beam (B&B) is a popular model in control system [1]. Through this model, algorithms are used to train students about control engineering. A fuzzy controller is used in [2]. This controller is designed by knowledge of experts in this B&B through a 49-rule fuzzy table. Inputs of fuzzy controller are error, which is between position of ball and expected position, and derivative of this error. In [4], B&B is regarded as a fuzzy T-S model. Through this model, a simple fuzzy model which satisfies Lyapunov's criteria is created. This controller proves its ability in stabilizing B&B through simulation. By this direction, no knowledge of experts is needed for designing control. However, there is no comparison between this method and other methods. Especially, the controller in **[4]** just guarantees the stability of system by mathematics. No calibration for control quality is considered.

LQR controllers are used in **[5]**, **[6]**. In these researches, both methods are proved to control B&B well. LQR controller is designed from linear dynamic equations of system. Some improvements are applied to get better control quality of system under LQR methods, such as, genetic algorithm (GA) for optimizing control matrixes of LQR method **[7]**. In that study, the axis of B&B is at one side of model. This structure is different form out B&B, which has axis at the middle of model. In this research, we propose a method of creating fuzzy controller from training data of system under LQR controller. By choosing a suitable set of data, fuzzy controller performs better than LQR controller. So, this fuzzy controller, which is created from LQR method can be regarded as a way to improve the ability of controller for B&B.

2. Dynamic Equations

In our B&B, a DC motor is located at the middle of the beam. Rotation of motor is transferred into the rotation of beam. Because of rotation of beam, a ball dimensionally moves on the beam. Thence, the rotation of axis of motor indirectly controls position of ball. The purpose of controlling is keeping position of ball at expected position. Usually, this expected position is at the middle of beam. Mathematical model of B&B is at [8]. From that study, the system parameters are shown in Tab. 1.

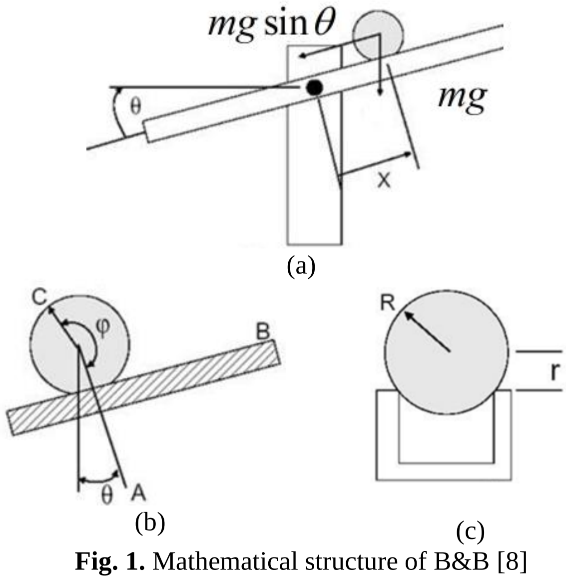

Fig. 1. Mathematical structure of B&B [8]

Tab. 1. System parameters

Parameters	Descriptions	Units
m_{ball}	Mass of ball	kg
m_{beam}	Mass of beam	kg

l	Length of beam	m
R	Radius of ball	m
b	Damping constant of friction of system	No unit
r	Distance in Fig. 1	m
g	Gravitation acceleration	m/s^2
K_t	Coefficient of motor (delivered by producer)	Nm/A

K_b	Coefficient of motor (delivered by producer)	V/(rad/s)
R_m	Coefficient of motor (delivered by producer)	ohm
C_m	Coefficient of motor (delivered by producer)	Nm/(rad/s)
J_m	Coefficient of motor (delivered by producer)	kgm^2

From [2], [3], dynamic equations of B&B are

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & \dfrac{-g}{1+\dfrac{2R^2}{5r^2}} & 0\\ 0 & 0 & 0 & 1\\ \dfrac{-m_{ball}g}{JB_m} & 0 & 0 & -(\dfrac{K^2{}_cK_tK_e}{JB_m}+\dfrac{b}{JB_m})\end{bmatrix}\begin{bmatrix}x\cos\theta\\ \dot{x}\\ \sin\theta\\ \dot{\theta}\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ \dfrac{K_cK_t}{JB_mR}\end{bmatrix}u \tag{1}$$

where: $JB_m = I_{beam}+K^2{}_cJ_m$; $I_{beam}=\frac{1}{12}m_{beam}l^2$ is inertial moment of beam (kgm^2); J_m is inertial moment of motor (kgm^2); u is voltage that is supplied to motor (volt).

Variables of system are

$$X=\begin{bmatrix}X_1 & X_2 & X_2 & X_2\end{bmatrix}^T=\begin{bmatrix}x & \dot{x} & \theta & \dot{\theta}\end{bmatrix}^T \tag{2}$$

Then, (1) is written as

$$\dot{X}=a(X)+f(X)u \tag{3}$$

where $a(X)=\begin{bmatrix}a_1(X) & a_2(X) & a_3(X) & a_4(X)\end{bmatrix}^T$ or $a=\begin{bmatrix}a_1 & a_2 & a_3 & a_4\end{bmatrix}^T$; $f(X)=\begin{bmatrix}0 & 0 & 0 & f_4(X)\end{bmatrix}^T=\begin{bmatrix}0 & 0 & 0 & f_4\end{bmatrix}^T$

If we assume that system just operates around working point

$$X_0=\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^T,\ u=0 \tag{4}$$

, nonlinear system in (3) can be accepted to be equivalent to a linear form as

$$\dot{X}=AX+Bu \tag{5}$$

Where A and B are linear matrixes.

Matrixes A and B are calculated as

$$A=\left.\begin{bmatrix}\frac{\partial a_1}{\partial X_1} & \frac{\partial a_1}{\partial X_2} & \frac{\partial a_1}{\partial X_3} & \frac{\partial a_1}{\partial X_4}\\ \frac{\partial a_2}{\partial X_1} & \frac{\partial a_2}{\partial X_2} & \frac{\partial a_2}{\partial X_3} & \frac{\partial a_2}{\partial X_4}\\ \frac{\partial a_3}{\partial X_1} & \frac{\partial a_3}{\partial X_2} & \frac{\partial a_3}{\partial X_3} & \frac{\partial a_3}{\partial X_4}\\ \frac{\partial a_4}{\partial X_1} & \frac{\partial a_4}{\partial X_2} & \frac{\partial a_4}{\partial X_3} & \frac{\partial a_4}{\partial X_4}\end{bmatrix}\right|_{\substack{X=X_0\\ u=0}};\ B=\left.\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ \frac{\partial f_4}{\partial X_1} & \frac{\partial f_4}{\partial X_2} & \frac{\partial f_4}{\partial X_3} & \frac{\partial f_4}{\partial X_4}\end{bmatrix}\right|_{\substack{X=X_0\\ u=0}} \tag{6}$$

In discrete-time, (5) can be described as

$$\dot{X}=A_dX+B_du \tag{7}$$

In Matlab, A_d and B_d is calculated by using command

$$[A_d, B_d]=c2d(A,B,T) \tag{8}$$

where: A, B is obtained from (5), T is sample-time

3. Controller Designing

3.1. LQR Controller

From Fig. 2, a structure of LQR controller is designed for B&B.

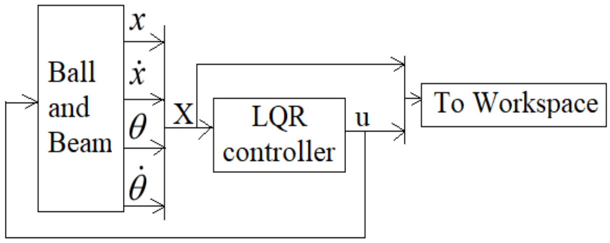

Fig. 2. LQR control structure for B&B

Control signal is voltage that is supplied to motor. This control signal is obtain from LQR method as

$$u = -KX \tag{9}$$

where $K = \begin{bmatrix} K_1 & K_2 & K_3 & K_4 \end{bmatrix}$ is control matrix

Matrix K is calculated from solving Ricatti equation. This work is compicated. Thence, Matlab software supply a demand to do this

$$K = dlqr\left(A_d, B_d, Q, R, T\right) \tag{10}$$

where A_d, B_d, T are obtained from (8)

3.2. Fuzzy Controller

After designing LQR controller in Section 3.1, we simulate it on Matlab. When the simulation shows the success of LQR in balancing system, we collect data $\left[x, \dot{x}, \theta, \dot{\theta}, u\right]$ through simulation time (as in Fig. 2). After each sample-time T, we obtain a sample of data. For example, if simulation time is 100s and sample-time is 0.01s, then, we have a set of 10001 samples. If simulation time is long, more data is collected and fuzzy block is more closed to LQR block. However, this action will make time of training longer. After getting data, toolbox ANFIS of Matlab is used for training a fuzzy block which imitate the LQR block. Display of ANFIS toolbox is shown in Fig. 5. Then, we obtain a four-input-one-output fuzzy controller. The rule table of this fuzzy controller has 81 rule.

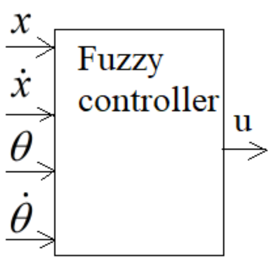

Fig. 3. Structure of fuzzy controller from ANFIS toolbox

We choose each input of fuzzy with memberships as in Fig. 4. Range of position of ball, velocity of ball, angle of beam, rotational velocity of beam, correspondingly, are [-0.27 0 0.27] (m), [-0.7 0 0.7] (m/s), [-0.4 0 0.4] (rad), [-0.1 0 0.1] (rad/s).

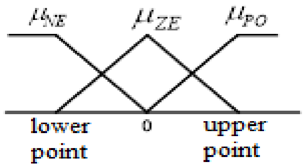

Fig. 4. Memberships of each input

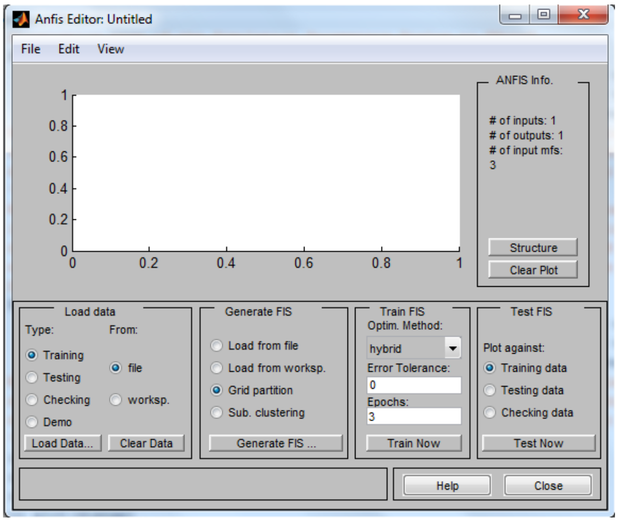

Fig. 5. Display of ANFIS toolbox

4. Simulation

System parameters in simulation in Tab. 1 have same values with real model in (11). These same values make simulation closed to experiment. In simulation, we examine response of system in three cases

- Case 1: Initial values of system are x = 0 (m), x_dot =0 (m/s), teta = 0 (rad), teta_dot = 0 (rad/s) (the B&B are balanced at equilibrium point)

- Case 2: Initial values of system are x = 0.1 (m), x_dot = 0.05 (m/s) , teta = 0.25 (rad), teta_dot = 0.05 (rad/s) (condition of B&B is near the equilibrium point)

- Case 3: Initial values of system are x = 0.22 (m), x_dot = 0.1 (m/s) , teta = 0.3 (rad), teta_dot = 0.1 (rad/s) (condition of B&B is far the equilibrium point)

Simulation results are shown from Fig. 6 to Fig. 10.

Case 1:

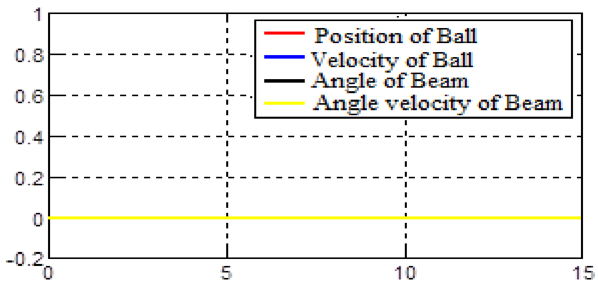

Fig. 6. Position of ball (m), velocity of ball (m/s), angle of beam (rad), angle velocity of beam (rad/s)

In this case, if there is no effect, system is kept exactly at equilibrium point.

Case 2:

In Fig. 7, settling time is 5s when system is under LQR and fuzzy. But, position of ball vibrates more strongly under LQR (0.12 rad) than fuzzy (0.003 rad). Also, in Fig. 8, angle of beam vibrates less under fuzzy controller (0.02 rad) than under LQR controller (0.15 rad). Thence, fuzzy method gives better simulation results than LQR method

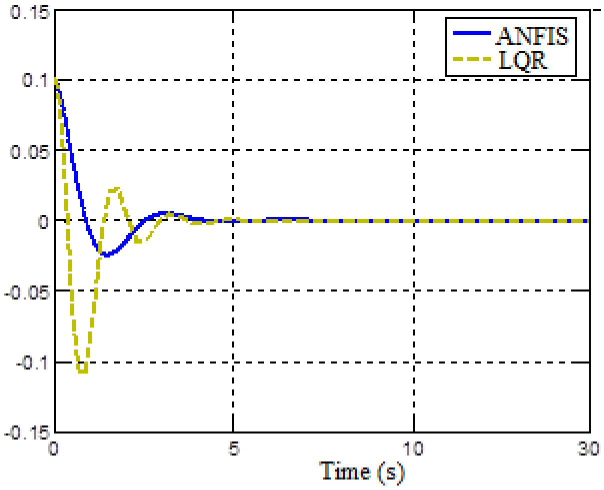

Fig. 7. Position of ball (m)

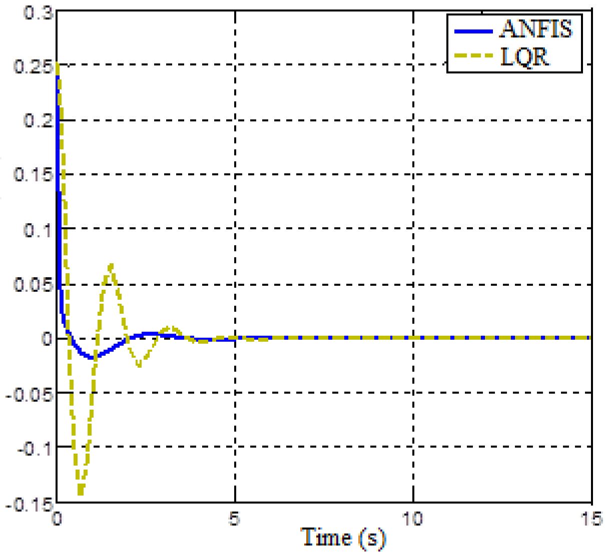

Fig. 8. Angle of beam (rad)

Case 3:

In Fig. 9, setling time in both methods is 5s. But, vibration of position of ball is smaller under fuzy method (0.05 rad) than under LQR method (0.08 rad). In Fig. 10, angle of beam vibrates 0.05 rad under fuzzy method and it vibrates 0.15 rad under LQR method. The vibration is smaller under fuzzy controller. Thence, responses of system under fuzzy controller show better quality than under LQR controller.

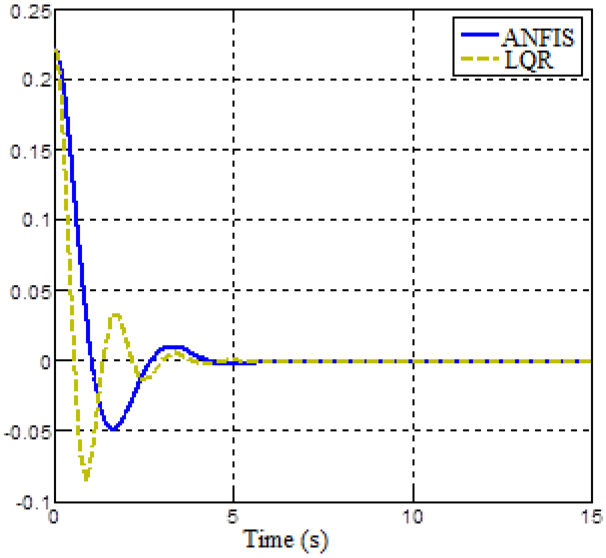

Fig. 9. Position of ball (m)

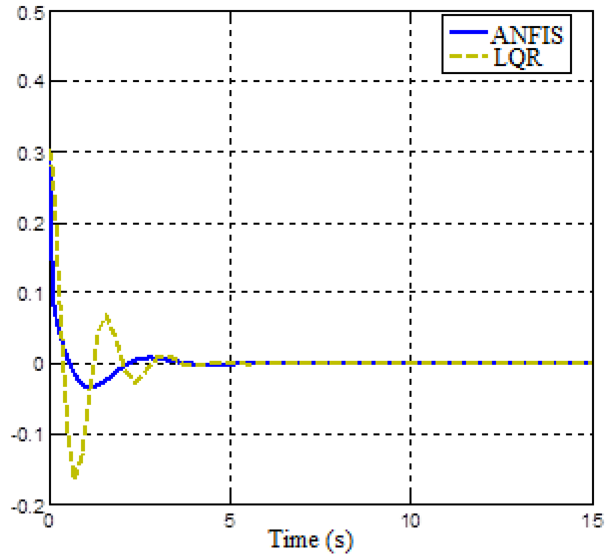

Fig. 10. Angle of beam (rad)

In 3 cases, fuzzy controller proves its better quality control than original LQR controller. Actually, suitable data is necessary. From our experience, we can try different simulation time and initial values of each input. There will be many results, which can be better or worse than original LQR. Only better fuzzy controller will be chosen. That fuzzy block is used for final simulation and experiment in this paper.

5. Experiment

5.1. Hardware Platform

We present an experimental model in Ho Chi Minh city University of Technology and Education (HCMUTE) in Fig. 11. In this model, DSP TMS320F28335 is used as control processor. The motion of ball is measured by resistance wire. Voltage is supplied to wire resistance. Thence, position of ball on beam is described by voltage that is measured at metal ball. Rotational angle of beam is measured by an encoder.

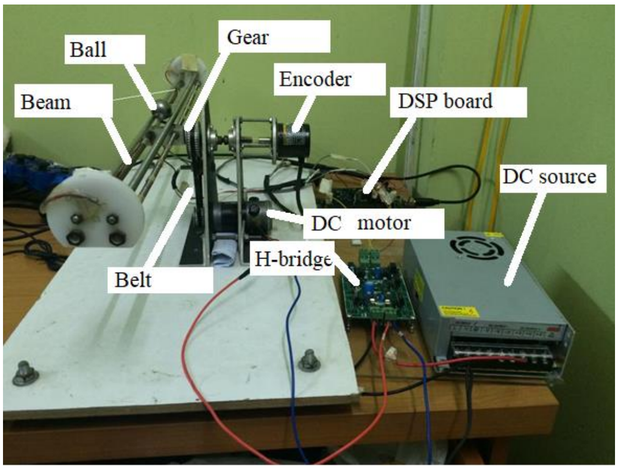

Fig. 11. Hardware platform of B&B in HCMUTE

The parameters of real model are measured and delivered by producer as

$$m_{ball}=0.015;\quad m_{beam}=0.23;\quad l=0.27;\quad R=0.005;\quad r=0.002;\quad g=9.81;\quad K_t=5.3e\text{-}3;\quad K_b=K_t;\quad R_m=2.7;\quad C_m=5e\text{-}4;\quad J_m=0.049e\text{-}4 \tag{11}$$

5.2. Experimental Results

Results are shown from Fig. 12 to Fig. 14. Both LQR and fuzzy methods describe their ability in balancing ball at equilibrium point well. In Fig. 12, fuzzy controller needs 950s (from second 200 to second 1150) to be settled, LQR controller needs 1000s to be settled (from second 150 to 1150). Also, system vibrates 0.2 rad (at second 210) under LQR method and 0.07 rad (at second 250) under fuzzy method. In Fig. 13, settling time is the same in both LQR and fuzzy methods (around 1000s). But, angle beam vibrates 0.12 rad under LQR method and 0.05 rad under fuzzy method. Then, fuzzy controller shows better quality control than LQR controller in experiments. In Fig. 14, the control signals of both methods are shown.

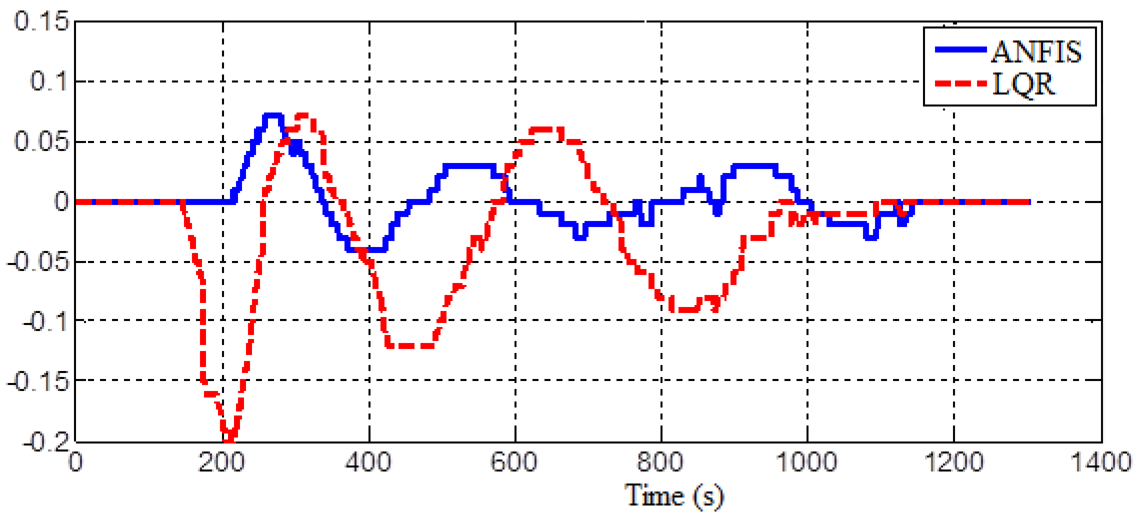

Fig. 12. Position of ball (m)

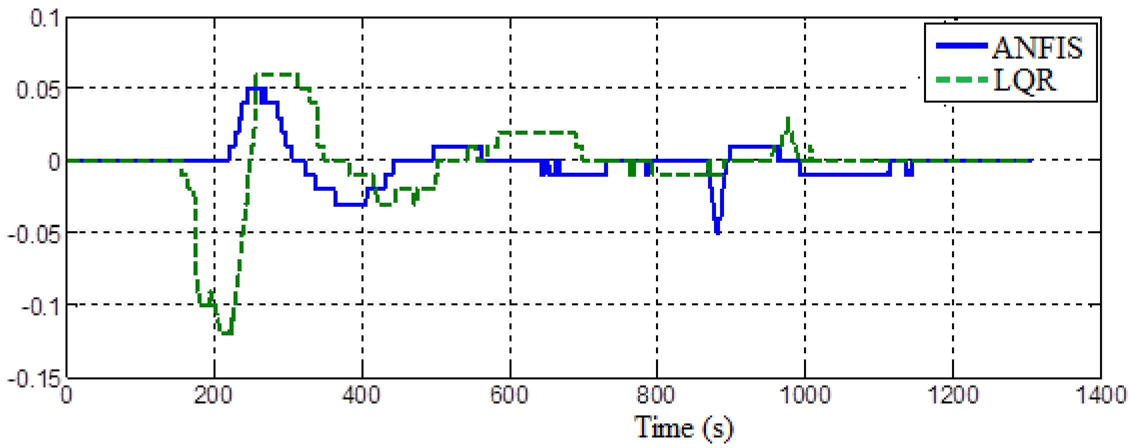

Fig. 13. Angle of beam (rad)

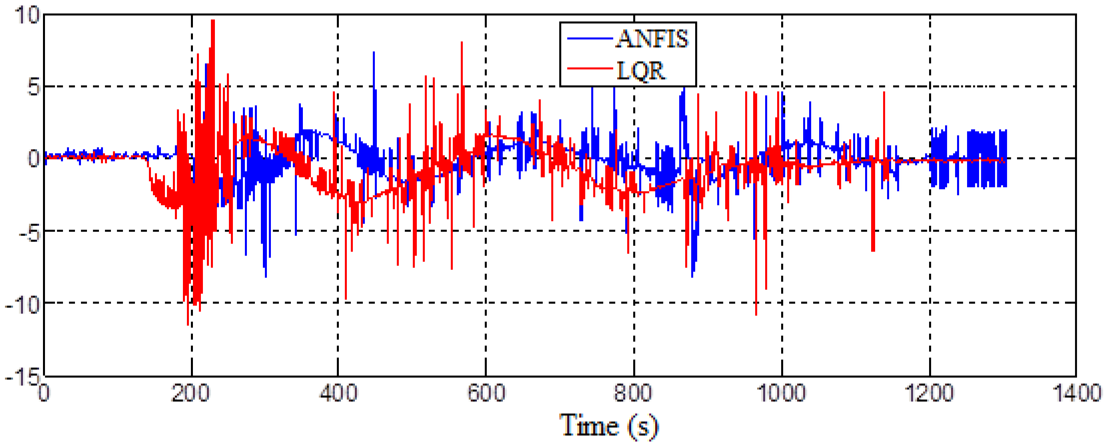

Fig. 14. Voltage on DC motor (V)

6. Conclusions

In our research, by using an LQR controller to stabilize B&B at equilibrium point in simulation, we collect data from this simulation and use that data to train a fuzzy controller by ANFIS toolbox. Through many sets of data, we obtain a fuzzy controller that shows better control quality than LQR controller. The comparison is described in both simulation and experiment. Our work can be regarded as a way to improve quality control of LQR controller by a toolbox ANFIS.

7. Acknowledgement

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8. References

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